MRCET CAMPUS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(AUTONOMOUS INSTITUTION – UGC, GOVT. OF INDIA)

Department of AERONAUTICAL ENGINEERING

SOLID MECHANICS

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SOLID MECHANICS

DIGITAL NOTES

B.TECH (R-22 Regulation)
(II YEAR – II SEM)
(2023-24)

DEPARTMENT AERONAUTCAL ENGINEERING

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

Recognized under 2(f) and 12 (B) of UGC ACT 1956

(Affiliated to JNTUH, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)

Maisammaguda, Dhulapally (Post Via. Hakimpet), Secunderabad – 500100, Telangana State, India

Department of AERONAUTICAL ENGIERRING

Vision

- Department of Aeronautical Engineering aims to be indispensable source in Aeronautical Engineering which has a zeal to provide the value driven platform for the students to acquire knowledge and empower themselves to shoulder higher responsibility in building a strong nation..

Mission

- The primary mission of the department is to promote engineering education and research. To strive consistently to provide quality education, keeping in pace with time and technology. Department passions to integrate the intellectual, spiritual, ethical, and social development of the students for shaping them into dynamic engineers.

QUALITY POLICY

- Impart up-to date knowledge to the students in Aeronautical area to make them quality engineers.Make the students experience the applications on quality equipment and tools.Provide systems, resources, and training opportunities to achieve continuous improvement.Maintain global standards in education, training, and services.

PROGRAM OUTCOMES
(PO's)

Engineering Graduates will be able to:

- Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
- Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
- Design / development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal and environmental considerations.
- Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
- Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
- The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
- Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
- Ethics: Apply ethical principles and commit to professional ethics and

responsibilities and norms of the engineering practice. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

- Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multi disciplinary environments.
- Life- long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM EDUCATIONAL OBJECTIVES – Aeronautical Engineering

- PEO1 (PROFESSIONALISM & CITIZENSHIP): To create and sustain a community of learning in which students acquire knowledge and learn to apply it professionally with due consideration for ethical, ecological and economic issues.

- PEO2 (TECHNICAL ACCOMPLISHMENTS): To provide knowledge based services to satisfy the needs of society and the industry by providing hands on experience in various technologies in core field.

- PEO3 (INVENTION, INNOVATION AND CREATIVITY): To make the students to design, experiment, analyze, and interpret in the core field with the help of other multi disciplinary concepts wherever applicable.

- PEO4 (PROFESSIONAL DEVELOPMENT): To educate the students to disseminate research findings with good soft skills and become a successful entrepreneur.

- PEO5 (HUMAN RESOURCE DEVELOPMENT): To graduate the students in building national capabilities in technology, education and research

PROGRAM SPECIFIC OUTCOMES – Aeronautical Engineering

- To mould students to become a professional with all necessary skills, personality and sound knowledge in basic and advance technological areas.

- To promote understanding of concepts and develop ability in design manufacture and maintenance of aircraft, aerospace vehicles and associated equipment and develop application capability of the concepts sciences to engineering design and processes.

- Understanding the current scenario in the field of aeronautics and acquire ability to apply knowledge of engineering, science and mathematics to design and conduct experiments in the field of Aeronautical Engineering.

- 4. To develop leadership skills in our students necessary to shape the social, intellectual, business and technical worlds.

MALLA REDDY COLLEGE OF ENGINEERING ANDTECHNOLOGY

II Year B. TECH – II- SEM ANE **L/T/P/C 4/-/-/4**

(R22A2108) SOLID MECHANICS

OBJECTIVES:

The course should enable the studentsto:

1. Acquire knowledge on the fundamental concepts ofstress and strain inmaterials
2. Describe beams and analyze Shear Force and Bending moments
3. Findthe slope and deflection in different types of beam
4. Determine critical loads of columns
5. Obtainknowledge on Strain Energy

Unit – I

Analysis of stress: Introduction to Solid Mechanics – Basic Concepts, Types of Stress, General State of Stress at a Point, State of stress at a point, Complimentary Shear stresses, Stresses on
Oblique planes
- Materials Subjected to pure shear, Material subjected to two mutually perpendicular direct
stresses.

Unit – II

Members Subjected to Flexural Loads: Geometric Forms of beams, Classifications of beams, statistically determinate Beams, Concept of Shear Force and bending moment in beams, Cantilever Beam and Simply Supported Beam- Shear Force and Bending Moment Diagrams, Simple Bending theory and Derivation of flexuralequation.

Unit – III

Deflection of beams: for a simply supported and Cantilever beam with problems using Double
Integration methodand Macaulay's method.

Unit – IV

Elastic stability of Columns: Euler's theory, Critical load determination of columns with different
endconstraints.

Unit – V

Theories of failures: Von-mises theory, octahedral shears distortion energy theory, Maximum
principle elastic strain theory, Maximum principle shear strain theory, Maximum shear stress
theory.

TEXT BOOKS:

1. Strength of Materials by R S Khurmi, S Chand and company Ltd
2. StrengthofMaterials by S Ramamrutam,DhanpatRai Publications

R18

Code No: **R18A2108**

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech II Semester Supplementary Examinations, April 2023

Mechanics of Solids

(AE)

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

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SECTION-I

1 Derive the expression for normal and tangential stresses on an oblique plane subjected to biaxial stresses. **[7M]**

Draw stress- strain diagram for mild steel specimen tested under uni-axial tension till fracture and mark all the salient points. **[7M]**

OR

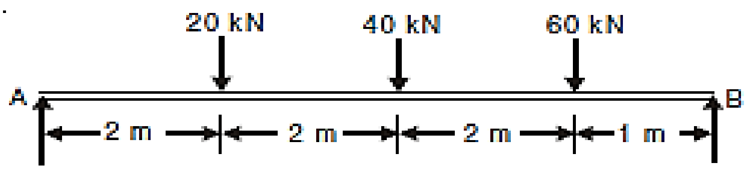
2 :ertain point in a strained material the principal stresses are 100 N/mm^2 and 40 N/mm^2 both tensile. Find the normal, tangential and resultant stresses across a plane through the point at 40^0 to the major principle plane, using Mohr's circle of stress. Check the answers Analytically **[14M]**

SECTION-II

3 :rive the bending equation from first principles. And find the maximum bending stress induced in the beam of rectangular cross section with 75 mm wide and 150 mm deep is simply supported over a span of 5 meters. if the beam is subjected to a UDL of 4.5 KN/m over entire span **[14M]**

OR

4 Plot shear and bending-moment diagrams for a simply supported beam with a point loads, shown in figure **[14M]**



SECTION-III

5 **a)** A simply supported beam of length L is subjected to a point load W at the mid span. Find the maximum deflection of the beam? **[7M]**

b) A simply supported beam of span 2.4 m is subjected to a central point load of 15 KN. What is the maximum slope and deflection at the center of the beam? Take EI [7M] for the beam as $6x10^{10}$ N-mm^2

OR

6 **a)** Derive an expression for the shear stress at any point in the cross-section of a beam. **[7M]**

b) A beam of rectangular cross section having width of 200 mm and height of 300 mm is subjected to a shear force of 25 kN. Find the value of maximum shear stress and draw shear stress distribution **[7M]**

SECTION-IV

7 **a)** Calculate the safe compressive load on a hollow cast iron column one end fixed and the other hinged of 150 mm external diameter, 100 mm internal diameter and 10 m in length. Use Euler's formula with a factor of safety of 5, and Take E= 95 KN/mm^2 **[7M]**

b) Derive crippling load by Euler's formula for a column having both ends of the column are pinned or hinged. **[7M]**

OR

8 **a)** Derive the Rankine's formula for both short and long column. **[7M]**

b) Find the crippling load by Rankine's formula for a hollow cylindrical steel column of 40 mm external diameter an 2.5 mm thick and the length of column is 2.5 m and hinged at its both ends take σ_c= 335 N/mm^2 and α=1/7500 **[7M]**

SECTION-V

9 Explain the following failure theories

i. the maximum principal stress theory **[4M]**
ii. the maximum principal strain theory **[5M]**
iii. the maximum shear stress theory **[5M]**

OR

10 A steel specimen is subjected to the following principal stresses (i) 125 N/mm^2 tensile (ii) 70 N/mm^2 tensile and 35 N/mm^2 compressive. If the proportionality limit for the steel specimen is 260 N/mm^2. Find the factor of safety according to (a) the maximum principal stress theory (b) the maximum principal strain theory (c) the maximum shear stress theory. Take Poisson's ratio μ = 0.3. **[14M]**

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Solid mechanics/strength of materials:-

UNIT-1: Analysis of stress:-

Introduction to solid mechanics - Basic concepts, types of stress, General state of stress at a point, state of stress at a point, complimentary shear stresses, stresses on oblique planes - Materials subjected to pure shear, Material subjected to two mutually perpendicular direct stresses.

Definition:-

In day to day work, an engineer comes across certain materials, i.e.. steel girders, angle irons, circular bars, cement etc.. which are used in his projects.

⇒ While selecting a suitable material for his project, an engineer is always interested to know its strength.

→ The strength of a meterial may be defined as ability, to resist its failure and behaviour, under the action of external forces.

⇒ It has been observed that, under the action of these forces, the material is first deformed and then its failure takes place.

A detailed study of forces and their effects, along with some suitable protective measures for the safe working conditions is known as strength of materials.

<u>Fundamental units:</u>

1. length
2. mass & time

<u>Derived units</u>

1. C.G.S units
2. S.I units
3. F.P.S units
4. M.K.S units

S.I units (international system of units)

Density (or mass density) kg/m^3

Force (in Newtons) N. $(=kg \cdot m/s^2)$

Pressure (in pascal) Pa $(=N/m^2 = 10^{-6} N/mm^2)$

Stress (in pascals) Pa $(=N/m^2 = 10^{-6} N/mm^2)$

Work done (in joules) J. (= N/m)

Power (in watts) W. (= J/s)

10^{12} Tera

10^9 mega

10^6

=> Whenever a force acts on a body, it undergoes some deformation and the molecules offer some resistance to the deformation. It will be interesting to known that when external force is removed, the force of resistance also vanishes; and the body springs back to its original position. But it is only possible, if the deformation, caused by the external force, is within a certain limit. Such a limit is called elastic limit.

=> The property of certain materials of returning back to their original position, after removing the external force, is known as elasticity.

=> A body is said to be perfectly elastic, if it returns back completely to its original shape and size, after the removal of external force

=> If the body does not return back completely to its original shape and size, after the removal of the external force, it is said to be partially elastic.

It has been observed that if the force, acting on a body, causes its deformation beyond the elastic limit, the body loses, to some extent, its property of

elasticity. If the external force, after
causing deformation beyond the elastic
limit, is completely removed, the body
will not return back to its original shape
and size. There will be some residual
deformation to the body, which will
remain permanently.

Stress:- → load per unit area.

Every material is elastic in nature. That
is why whenever some external system of
forces acts on a body, it undergoes some
deformation. As the body undergoes defor-
-ation. This resistance per unit area to
deformation, is known as stress. Mathemati-
-cally stress may be defined as the force
per unit area. i.e stress

$$\sigma = P/A$$

P = load or force acting on the body, and

A = cross-sectional area of the body

SI unit - Pascal.

Strain:- (load per unit length)

If an body undergoes some deformation.
This deformation per unit length is known
as strain. Mathematically strain may be

defined as the deformation per unit length.

$$\varepsilon = \frac{\delta l}{l} \qquad \delta l = \varepsilon \cdot l$$

δl = change of length of the body.

l = original length of the body.

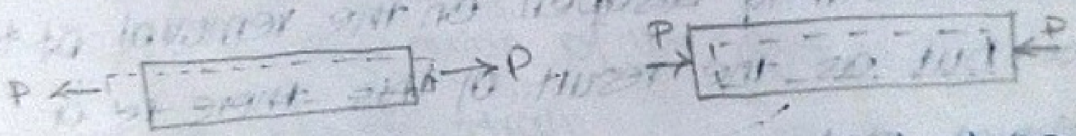
Types of stress:-

Though there are many types of stresses from the subject point of view.

1. Tensile stress 2. compressive stress

Tensile stress:-

If the force is applied on both the end in oppsite direction. due to the applied force it undergoes some changes.



When a section is subjected to two equal and oppsite pulls and the body tends to increases its length. as shown in fig. the stress induced is called tensile stress. The corresponding strain is called tensile strain.

compressive stress:-

When the force are applied on the both side in the same direction. due that force. it changes the shape of the object.

Elastic limit:-

When ever some external system of forces act on a body it undergoes some deformation if the external forces, causing deformation, are removed the body. spring back to its original position.

It has been found that for a given section there is a limiting value of force upto and within which the deformation entirely disapears on the removal of force. The value of intensity of stress corresponding to this limiting force is called elastic limit of the material.

Beyond the elastic limit the material gets into plastic stage and in this stage the deformation does not entirely disapers on the removal of the force. but as the result of this there is a residual deformation even after the removal of the force.

Hooke's law:-

It states that when a material is loaded with-in it's elastic limit the stress $\propto$ strain.

Mathematically:-

$$\frac{\text{stress}}{\text{strain}} = E = \text{constant}.$$

$$\sigma \propto e \Rightarrow E \times e.$$

or

$$E = \frac{\sigma}{e}.$$

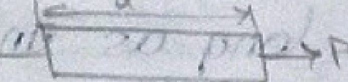
Where as

$\sigma \rightarrow$ stress $\quad e \rightarrow$ strain.

e = a constant proportionality, known as modulus of elasticity (or) young's modulus

Deformation of a body due to force acting on it:-

consider a body subjected to tensile stress. let P load-force acting on a body, 'l' length



of the body, 'A' cross-sectional area of the body.

σ = stress induced in the body,

e = modulus of elasticity for the material of the body.

ε = strain

δl = Deformation of the body.

W.K.T stress, strain

$$\sigma = P/A \cdot \quad \varepsilon = \frac{\sigma}{e} = P/_{AE} \quad \left(\sigma = P/_{A}\right)$$

Now the deformation is,

$$\frac{\delta l}{l} = \varepsilon \quad \delta l = \varepsilon l = \frac{\sigma l}{E} = \frac{Pl}{AE}$$

Problems:-

1) A steel rod 1m long and 20x20mm cross section is subjected to a tensile force of 40kN. determine the elongation of the rod. for the rod material is GPA (e) a parall

Sol:- length $l = 1m = 1\times10^3$ mm.

area $(A) = 20\times20mm = 400mm^2$

Force $= 40kN = 40\times10^3$

Modules of rigidity $(E) = 200\times10^3$ N/mm^2

W.K.that $1GPa = 1000N/mm^2$

the elongation of the rod.

$$\delta l = \frac{Pl}{AE} = \frac{40\times10^3\times1\times10^3}{400\times200\times10^3} = 0.5\ mm$$

2) A hallow cylinder 2m long as an outside diameter of 50mm and inside diameter of 30mm if the cylinder is caring a load of 25kN. find the stress in the cylinder and also find the deformation of the cylinder, if the value of modules of elasticity (E) is 100×10^9 Pascal.

Sol:- length $l = 2m = 2\times10^3$ mm

outside (dia) = 50mm, inside (dia) = 30mm.

force $= 25kN = 25\times10^3$ N/mm

$E = 100GPa = 100\times10^3$ N/mm^2

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(50^2 - 30^2)$$

$$= 1256\ mm^2$$

Stress:-

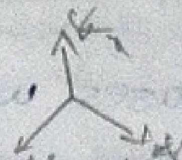
$$\sigma = P/A = \frac{25\times10^3}{1256} = 19.904$$

Deformation:-

$$\delta l = \frac{Pl}{AE} = \frac{25\times10^3 \times 2\times10^3}{1256\cdot\times100\times10^3} = 0.398 \approx \text{umm}$$

Principal Planes:-

Principal stress:-



at any point in a strained material. there are three planes, mutually perpendiculars to each other, which carried direct stress only. and shear stress. are known as principal planes.

Principal stress:-

The magnitude of direct stress across a Principal plane is known as principal stress.

Methods for the stress on a oblique section of a body

The following two methods for the determination of the stress on an oblique section of a strained body. are

i) analytical & graphysical method.

analytical method. for the stress on an oblique section of a body.

The analytical method for the determination of stress on an oblique section in the following cases.

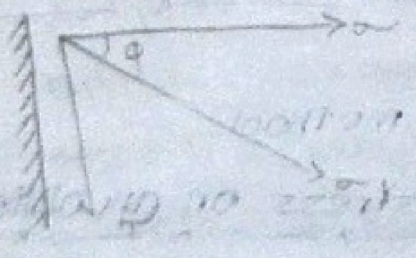
case(i):- A body subjected to direct stress. in only one plane.

ii) A body subjected to direct stress in two mutually perpendicular directions.

Oblique stress :- (obliquity, slant)

In engineering cases we don't have, pure, compression (or) pure tension (or) pure shear but combination of two stress. Oblique stress comes into picture. When we have a combination direct stress and shear stress.

Direct stress can be tensile stress (or) compressive stress, denoted by sigma and shear stress. denoted by τ. The combination of sigma (σ) and τ produce a resultant. which is neither normal (or) parallel to the surface. but it is in an angle and this angle will bring the obliquity of the stresses.



$$\sigma r = \sqrt{\sigma^2 + \tau^2}$$

$$\tan\phi = \frac{\tau}{\sigma}$$

$$\phi = \tan^{-1}\left(\frac{\tau}{\sigma}\right)$$

Sign conventions :-

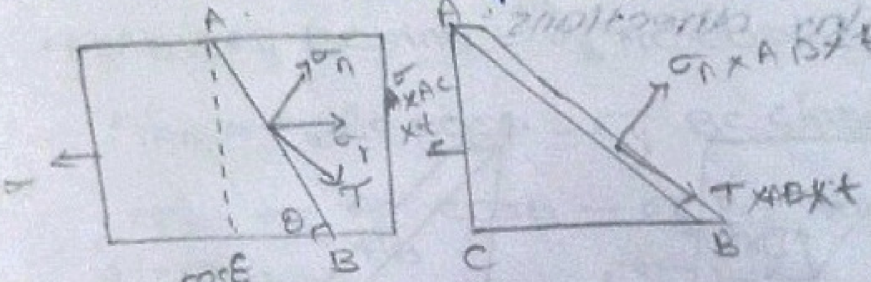
i) all tensile stress, & strains are taken as positive and compressive stress are negative.

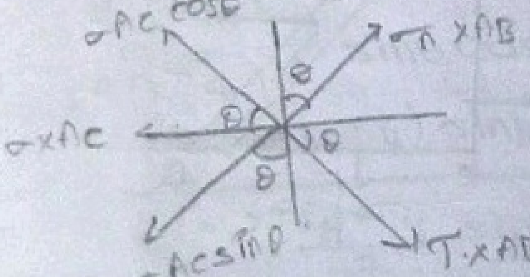
or shear stress which rotate the element clockwise. is positive. where as the shear stress. which. rotate the element anticlockwise is Negative.

case:-1

Stress on oblique section of a body subjected to direct stress in one plane:-

consider a rectangular body of uniform





cross sectional area and unit thickness subject to a direct tensile stress along 'x-x' axis. let us consider a oblique section AB inclined with x-x axis

$$\sigma_n \times AB = \sigma \times AC \times \sin\theta$$

$$\sigma_n = \sigma \times \frac{AC}{AB} \sin\theta$$

$$\sigma_n = \sigma \times \sin\theta \times \sin\theta$$

$$\boxed{\sigma_n = \sigma \times \sin^2\theta} \leftarrow (1)$$

Now $\tau \times AB = \sigma \times AC \times \cos\theta$

$$\tau = \sigma \times \frac{AC}{AB} \times \cos\theta \quad (\sin 2\theta = 2\sin\theta\cos\theta)$$

$$\tau = \sigma \times \sin\theta \times \cos\theta$$

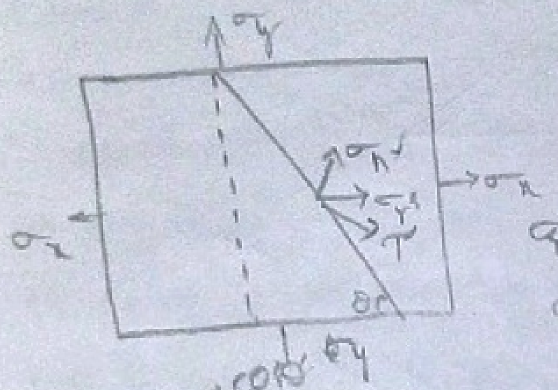
$$\boxed{\tau = \sigma \times \frac{\sin 2\theta}{2}}$$

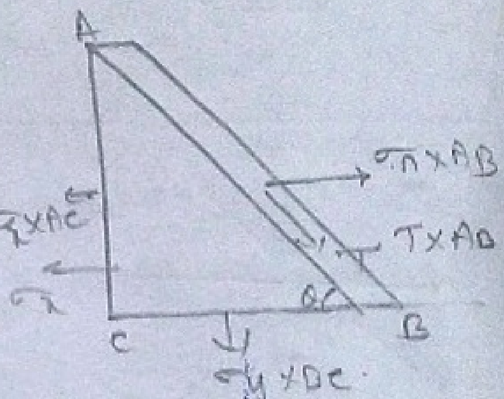
σ_n = maximum when $\theta = 90^\circ$

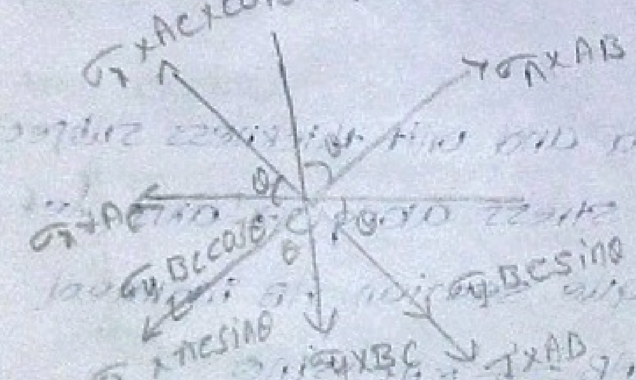
τ = maximum when $\theta = 45^\circ$

case ii :-

Stressel on oblique section of a body subjected to direct stress in two mutually perpendicular directions.







consider a body of uniform cross sectional area and unit thickness subjected to tensile stress in two mutually perpendicular in the direction of x-x and y-y. let us consider oblique stress inclined with x-x axis

$$\sigma_n AB = \sigma_x AC\sin\theta + \sigma_y BC\cos\theta$$

$$\sigma_n = \sigma_x \cdot \frac{AC}{AB} \sin\theta + \sigma_y \cdot \frac{BC}{AB} \cos\theta$$

$$\sigma_n = \sigma_x \times \sin\theta \sin\theta + \sigma_y \cos\theta \cos\theta$$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta$$

$$\boxed{= \sigma_x \left(\frac{1-\cos 2\theta}{2}\right) + \sigma_y \cdot \left(\frac{1+\cos 2\theta}{2}\right)}$$

$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta$$

$$\sigma_y = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta$$

$$\tau \times AB + \sigma_y \cdot BC \sin\theta = \sigma_x AC \cos\theta$$

$$\tau AB = \sigma_x AC \cos\theta - \sigma_y BC \sin\theta$$ 21-4-50

$$\tau = \sigma_x \frac{AC}{AB}\cos\theta - \sigma_y \frac{BC}{AB}\sin\theta$$

$$\tau = \sigma_x \sin\theta\cos\theta - \sigma_y \cos\theta \sin\theta$$

$$\tau = (\sigma_x - \sigma_y)\sin\theta\cos\theta$$

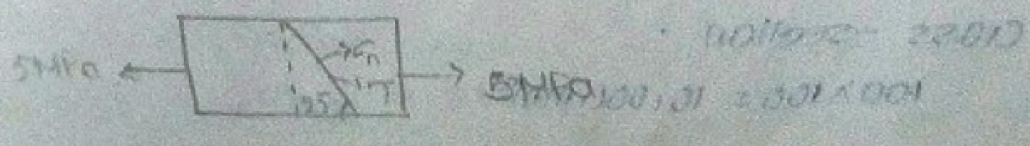
$$\boxed{\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta}$$

shear stress will be maximum when $\theta = 45°$

Problems based on the case i & case ii:

1) A wooden bar is subjected to a tensile stress of 5MPa. What will be the values of normal and shear stress across the section, which make an angle of 25° with the direction of tensile stress.

Sol:



$$\sigma_n = \sigma \sin^2\theta$$

$$= \sigma\left(\frac{1-\cos 2\theta}{2}\right)$$

$$= \frac{\sigma}{2} - \frac{\sigma\cos 2\theta}{2}$$

$$= 5/2 - \frac{5\cos 2(25)}{2}$$

$$\sigma_n = 5/2 - 5/2 \cos 50$$

$$\sigma_n = 0.893\text{MPa}$$

shear stress:

$$\tau = \frac{\sigma}{2}\sin 2\theta$$

$$= \frac{\sigma \sin 2(25)}{2}$$

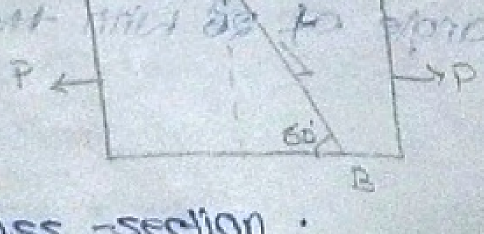
$$= \frac{\sigma}{2}\sin 50$$

$$= 5/2 \sin 50$$

$$= 1.915\text{MPa}$$

Q) Two wooden piece 100x100mm in cross-section are joined together along a line. AB as shown in below fig. find the maximum force which can be applied if the shear stress along the joint AB is 1.8MPa.

Sol:



cross-section.

$100 \times 100 = 10{,}000\text{mm}^2$

$\theta = 30°$

$\tau = 1.3\,MPa = 1.3 \times 1\,N/mm^2$

We know,

$$\tau = \frac{\sigma}{2}\sin 2\theta$$

$$1.3 = \frac{\sigma}{2}\sin 2(30)$$

$$1.3 = \frac{\sigma}{2}\sin(60)$$

$$\sigma = 3.00\,N/mm^2$$

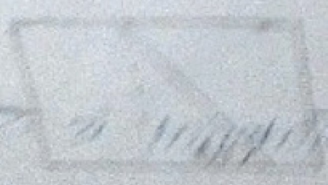
∴ Maximum axial force which can be applied

is $P = \sigma \times A$ $\left(\because \sigma = \frac{P}{A}\right)$

$= 3.0\,kN$

$= 3000\,N = 3\,kN$

3) A tension member is formed by connecting two wooden members 200mm×100mm as shown in below fig. Determine the safe value (minimum value) of the force P if permissible normal and shear stress in the joint are 0.5 MPa and 1.25 MPa respectively.



Sol:- Cross section 200mm × 100mm

$= 20{,}000\,mm^2$

$\theta = 60°$

$\sigma = 0.5\,MPa$

$\tau = 1.25\,MPa$

$\theta = 60°$

$\tau = 1.3 MPa \rightarrow 1.3 N/mm^2$

W.k.T,

$$\tau = \frac{\sigma}{2} \sin 2\theta$$

$$1.3 = \frac{\sigma}{2} \sin 2(60)$$

$$1.3 = \frac{\sigma}{2} \sin(120)$$

$$\sigma = 3.00 N/mm^2$$

∴ Maximum axil force which can be applied

is $P = \sigma \times A$ $\quad (\because \sigma = P/A)$

$= 3.0 \times N$

$= 30000 N = 30 KN$

3) A tension member is formed by connecting two wooden frame members. 200mm×100mm as shown in below fig. determine the safe value (minimum value) of the force P if permisible normal and shear stress in the joint are 0.5MPa and 1.25MPa respectively

Sol: Cross section 200mm×100mm

$= 20,000 mm^2$

$\theta = 60°$

$\sigma_n = 0.5 MPa$

$\tau = 1.25 MPa$

W.K.T

$\sigma_n = \sigma \sin^2\theta$

$\sigma_n = \sigma\left(\frac{1-\cos 2\theta}{2}\right)$

$= \frac{\sigma}{2} - \frac{\sigma \cos 2\theta}{2}$

$0.5 = \frac{\sigma}{2} - \frac{\sigma \cos(120)}{2}$

$0.5 = \frac{\sigma}{2}\left(1 - \frac{\cos(120)}{2}\right)$

$\sigma = 0.67\ N/mm^2$

$\tau = \frac{\sigma}{2} \sin 2\theta$

$1.25 = \frac{\sigma}{2} \sin(120)$

$\sigma = 2.886\ N/mm^2$

From the above two values of σ we found that safe stress is the least of two value $0.67 N/mm^2$.

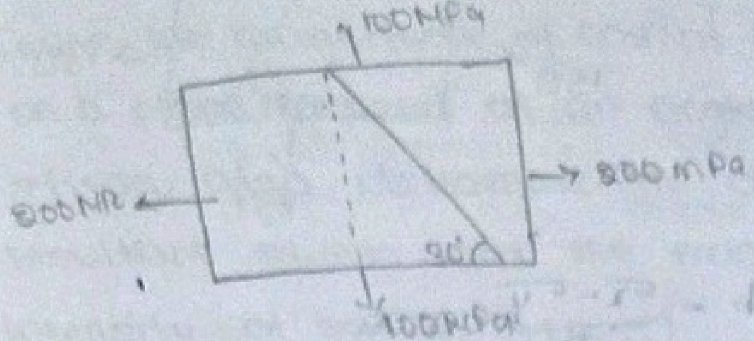
$\therefore P = \sigma A$

$= 0.67 \times 20,000$

$P = 13.4\ KN$

Case ii problems:

1) A point in a strained material is subjected to two mutually perpendicular tensile stress are 200MPa and 100MPa, determine the intensity of normal, shear and resultant stress on a planed inclined at 30° with the access of major tensile stress.



$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta$$

$$= \frac{200}{2} - \frac{200}{2}\cos(60) + \frac{100}{2} + \frac{100}{2}\cos(60)$$

$= \cancel{128.05\,MPa}$ $125\,MPa$

shear stress

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

$$= \frac{200-100}{2}\sin(60)$$

$= 43.301\ MPa$

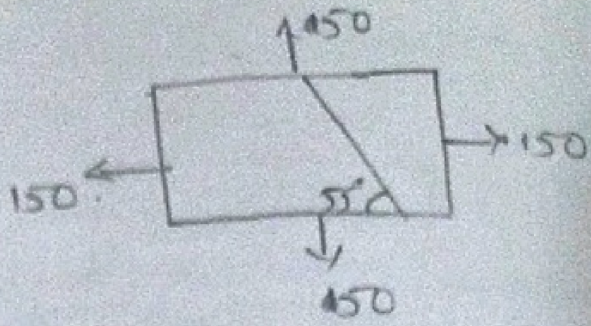
resultant

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

$\sigma_r = \cancel{128.76}\ \cancel{128.8}\ MPa$
$132.3\ MPa$

* The stress at a point of machine component. are 150MPa and 50MPa both tensile find the intensity of normal, shear and resultant stresses on a Plane inclined at an angle of 55° with the axis of major tensile stress. also find the magnitude the max shear, stress in the

component.



Now

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta$$

$$= \frac{150+50}{2} - \frac{150-50}{2}\cos 2\times 55$$

$$= 100-50\cos(110)$$

$$\boxed{\sigma_n = 117.10\,MPa}$$

$$\tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta$$

$$= \frac{150-50}{2}\sin 2\times 55$$

$$= 50\sin 2\times 55$$

$$\boxed{\tau = 46.98 \simeq 47\,MPa}$$

Resultant stress

$$\sigma_r = \sqrt{(\sigma_n)^2+(\tau)^2}$$

$$= \sqrt{(117.1)^2+(47)^2}$$

$$\boxed{\sigma_r = 126.18\,MPa}$$

Magnitude of max shear stress is

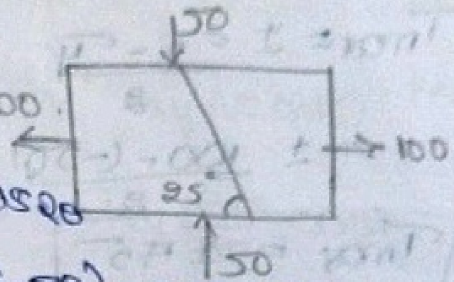
$$\tau_{max} = \pm\frac{\sigma_x-\sigma_y}{2}$$

$$= \pm\frac{150-50}{2}$$

$$\tau_{max} = \pm 50.$$

* The stress at a point in a component are 100MPa (tensile) and 50MPa (compressive) determine the magnitude of normal and shear stress on a plane inclined at an angle 25° with tensile stress, also determine the direction of resultant stress, and the magnitude of max. intensity of stress stress?

Normal force



$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta$$

$$= \frac{100+(-50)}{2} - \frac{100-(-50)}{2}\cos 2\times 25$$

$$= \frac{100-50}{2} - \frac{100+50}{2}\cos 50°$$

$$= 25 - 75\cos 50$$

$$\boxed{\sigma_n = -23.209\text{MPa}}$$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

$$= \frac{100-(-50)}{2}\sin 2\times 25$$

$$= 75\sin 50$$

$$\boxed{\tau = 57.45\text{ MPa}}$$

direction of resultant stress

$$\sigma_r = \sqrt{\sigma_n^2+\tau^2}$$

$$= \sqrt{(-23.209)^2+(57.45)^2}$$

$$\boxed{\sigma_r = 61.96}$$

θ, is the angle which resultant stress make with x-x axis

$$\tan\theta = \frac{\tau}{\sigma_n}$$

$$\tan\theta = \frac{57.45}{-23.209} \quad \boxed{\theta = -68.001}$$

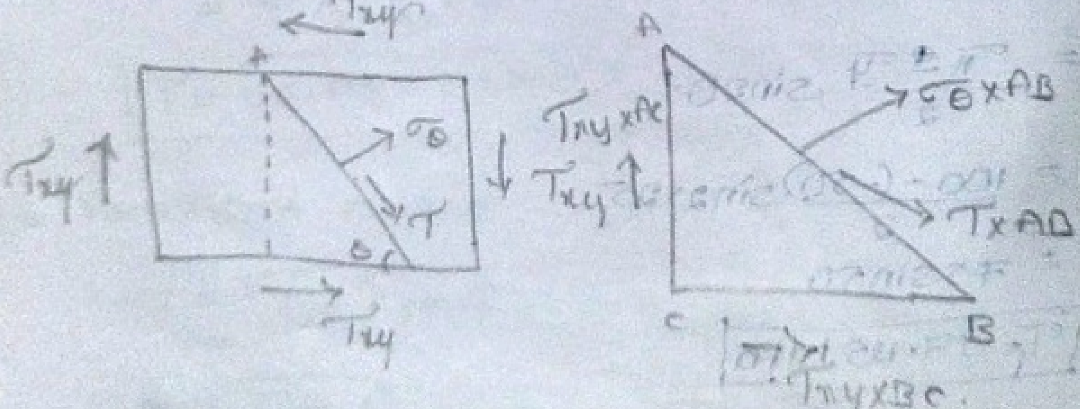
Magnitude of shear stress τ_{max}

$$\tau_{max} = \pm \frac{\sigma_x - \sigma_y}{2}$$

$$= \pm \frac{100-(-50)}{2}$$

$$\boxed{\tau_{max} = \pm 75}$$

Case-8:- Stress on a oblique section of a body subjected to a body simple shear stresses



consider a rectangular body of uniform crossection area and unit thickness subjected to a +ve shear stress along x-x axis as per the pirciple of simple shear stress the face BC experience and anti clock wise shear stress.

$\sigma_\theta AB - \tau_{xy} BC \sin\theta - \tau_{xy} AC \cos\theta = 0.$

$\sigma_\theta = \tau_{xy} \frac{BC}{AB} \sin\theta + \tau_{xy} \sin\theta \cos\theta$

$= 2\tau_{xy} \sin\theta\cos\theta.$

$= \cancel{2} \tau_{xy} \times \frac{\sin 2\theta}{\cancel{2}}$

$$\boxed{\sigma_\theta = \tau_{xy} \times \sin 2\theta.}$$

$\tau_\theta AB + \tau_{xy} BC \cos\theta = \tau_{xy} AC \sin\theta$

$\tau_\theta = \tau_{xy} \frac{AC}{AB} \sin\theta - \tau_{xy} \frac{BC}{AB} \cos\theta$

$= \tau_{xy} \left(\frac{1-\cos 2\theta}{2}\right) - \tau_{xy} \left(\frac{1+\cos 2\theta}{2}\right)$

$= \cancel{\frac{\tau_{xy}}{2}} - \frac{\tau_{xy}}{2}\cos 2\theta - \cancel{\frac{\tau_{xy}}{2}} - \tau_{xy} \frac{\cos 2\theta}{2}$

$= -\tau_{xy} \frac{\cos 2\theta}{2} - \tau_{xy} \frac{\cos 2\theta}{2}$

$= -\cancel{2} \frac{\tau_{xy} \cos 2\theta}{\cancel{2}}$

$$\boxed{\tau_\theta = -\tau_{xy} \cos 2\theta}$$

Material properties :-

Mechanical properties of a material :-

Mechanical properties are more important properties of material from the engineering point of view. in selecting them for the design purposes. mechanical properties of the material describe their behaviour under mechanical usage.

an engineering must a have considerable knowledge of materials and their properties. the most important mechanical properties of materials are strength, stiffness, elasticity, plasticity, Brittleness, mallability, tuffness, hardness, resilience, creep, etc.

1) strength:
It is the ability of the material to resist to external load, applied force.

2) stiffness:
The ability of a material to resist deformation under stress.
→ The modules of elasticity is the measure of stiffness.

3) elasticity:
It is the property of the material, to regain its original shape after deformation, when external forces are removed.
It property desired materials used in tools and machine

4) Plasticity:
It is the property of a material which retain the deformation produced under load permanently is known as plasticity.
→ This property of the material is necessary for forging in stamping image on coins, and in ornamental work.

5) ductility:-

It is the ability of a material to under go deformation under tensile load due to this property material can be drawn into wires. This property is usually measured by percentage of elongation and reduction in area.

6) Bitterness:-

It is the property of the material opposite to the ductility. This the property of material with breakdown little perment destroy

7) malleability:-

It is the ability of material under go compression due to this property metals can be rolled or hammered into thin sheets

8) ~~tuffness~~ toughness:-

It is the property of material to resist facture due to high impact load like hammer gloes

It is measured by the amount of energy of unit volume of the material has observed after been stressed upto the point of facture. The tuffness of material decrease when it is heated.

It property is desirable in parts subjected to shock and impact load

hardness -

The ability of to resist wear scratching, deformation and machining.

It is also mean the ability of metal to cut another metal.

resilience to absorve energy and to re...
It is the property of the material to resi... impact load.
It measured by the amount of energy absorbed per unit volume within elastic limits. This property is essial for spring materials.

creep:

It is a property to undergo slow and progressive deformation, over a period of time under constant stress.

A material subjected to constant tensile load elevated temperature will creep under a time deformation this property is consider in design internal combustion engines, boilers and turbine.

Types of stress:

Various types of stress may be classified
a. simple or direct stress
a) tensile stress.
b) compressive stress.
c) shear stress.

2) In direct stress.
a) bending. stress
b) trosion stress
c) combined stress.
any possible comination of 1 & 2

The tensile stress:-

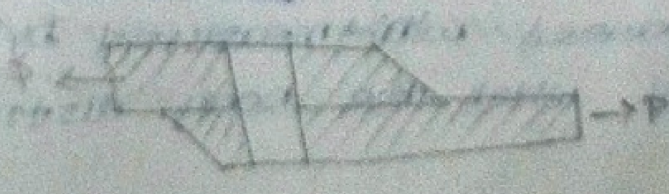
The stress induced in a body when subject-ed to two equal opposite pulls as a result of which that increase in length. is known as tensile stress

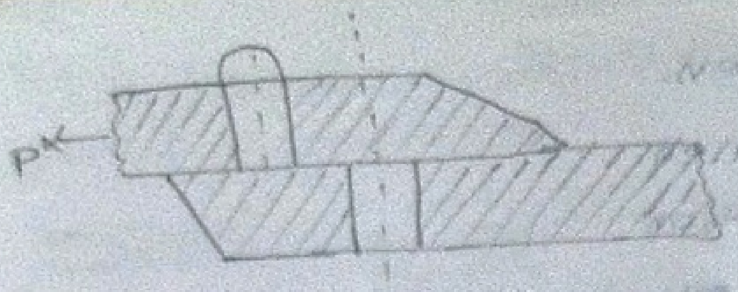
compressive stress:-

The compressive stress induced in a body. when subjected to two equal and opposite pushes. as a result of which their is a decrease in length. is known as compressive stress

shear stress:-

The stress induced in a body when subjected two equal and opposite forces which are acting tangentially across resisting section as a result of which the body tends to shear of across the section is known as shear stress.





shear stress.

$$\text{shear stress} = \frac{\text{shear resistance}}{\text{shear Area}} = P/A.$$

strain:-

tensile strain:-

When a body is subjected to tensile load the ratio of increase in length through the orginal length is called tensile strain.

$$\varepsilon = \frac{\text{Increase in length}}{\text{orginal length.}}$$

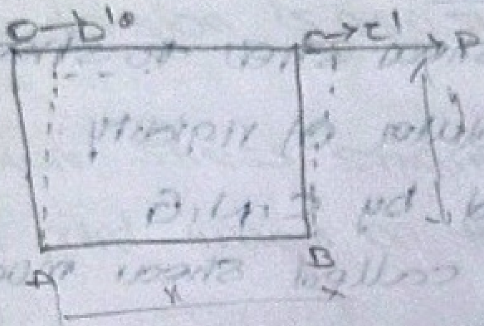
compressive strain:-

When a body is subjected to compressive load the ratio of decrease in length through the orginal length is called compressive strain

$$\varepsilon = \frac{\text{decrease in length}}{\text{orginal length}}$$

shear strain:-

In case of shearing load, shear strain will be produced which measured by the angle through which the body distroted consider a rectangular block as shown in fig a.b.c.d. fixed at one face. subjected to a tensile force 'p' along the top face of the block after application of force it distort through an angle 'ɸ' and occuied a new position ABc'.d'. The shear strain is given by $\frac{cc'}{Bc} = \frac{DD'}{AD} = x/y$



Volumetric strain:-

The ratio of change in volume of the body to original volume of the body is known as volumetric strain.

$$\varepsilon_v = \frac{\text{change in volume}}{\text{original volume}} \quad \frac{\delta v}{v}$$

elastic constant:-

depending on a nature of strain the elastic constant are

① modula of elasticity.

② modula of rigidity.

③ bulk modules.

Modulus of elasticity are also called young's modulus, the ratio of tensile stress and tensile strain (or) compressive stress and compressive strain is called modulus of elasticity (E)

$$E = \frac{\text{stress}}{\text{strain}} = \sigma/\varepsilon$$

Modulus of rigidity:-

The ratio of shear stress to shear strain is called modulus of rigidity.

It is denoted by C, N, G

It is also called shear moduli of elasti

Modulus of rigity (G) $\frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$

Bulk Modulus:-

When a body is subjected to three mutual perpendicular stresses, the ratio of direct stress to the volumetric strain.

The ratio of

It is denoted by (K) = $\frac{\text{direct stress}}{\text{volumetric strain}}$

$$\boxed{K = \frac{\sigma}{\varepsilon_v}}$$

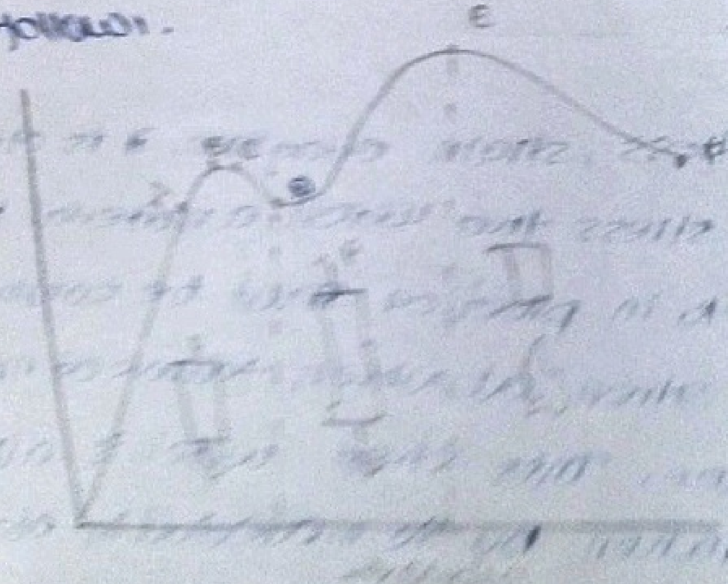
The S.I units of E, G, K are N/m^2 (or) N/mm^2.

Stress-strain diagram:

Stress-strain diagram for ductile material:

Mild steel is most common used ductile material in this test a mild steel specian under tensile load is tested in universal testing machine upto distruction, in [illegible]. The load applied is indicated on a [illegible] and the tension is measured by [illegible] meter.

fig shown stress-strain for mild steel the stell point noted on the curve are as follow.



A = Proportional limit

B = elastic limit

C = upper yield point

D = lower yield point

E = ultimate stress

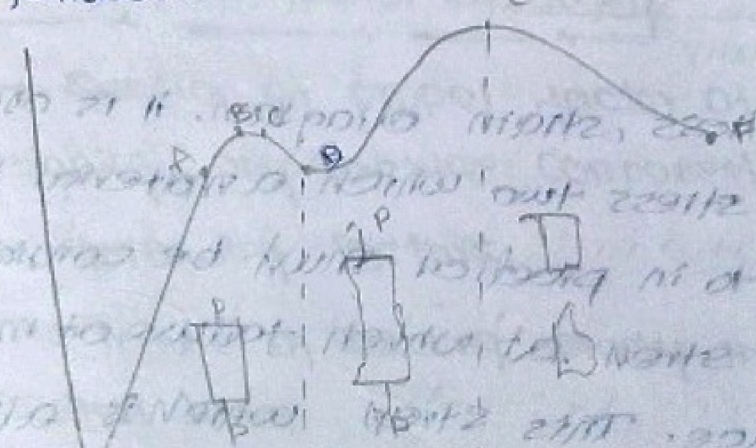
F = breaking stress

Stress Strain diagram:-

Stress strain diagram for ductile material:-

Mildsteel is most common used ductile material in this test a mild steel specian under tensile load is tested in universal testing machine. upto distruction. in UTM The load applied is indicated on a dile and the tension is measured by extensio meter.

fig shown stress, strain for mild steel the silent point mared on the curve are as follow.



A→ Perposional limit

B is elastic limit

C is yelid positionpoint

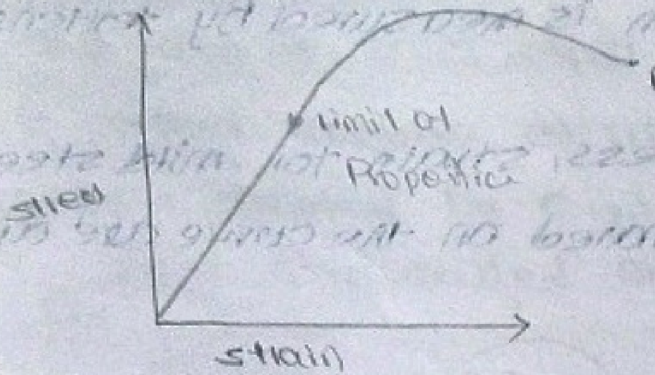
D is plastic limit

E is Ultimate stress.

F is Breaking stress

stress, strain diagram for a Buttile material:-

like ~~cc~~ cast iron shown in fig. there is a little elongation and reduction in area of specimen for such material the stright line. The yeild point is not marked at all. The stright line position of diagram is very small.



from the stress, strain diagram. it is obeying the greatest stress two which a material is subjected to in practice must be convalue to ~~ad~~ ultimate stress at which failure of material st takes place. This stress which is allowed to under taken by the material in design is called the working stress (or) design stress.

The ratio of ultimate stress to working stress is called factor of ~~saffy~~ safety

∴ factor of saffy = $\dfrac{\text{Ultimate stress}}{\text{Working stress (or) design stress}}$

Note:-

In case of ductile material, eg. mild steel where the yeild point is clearly defined. the factor of safety is based on yeild point stress in such cases.

factor of safety = $\frac{\text{yeild point stress}}{\text{working stress}}$.

The value of factor safety varies from three in case of steel, two as high as 20 in case of timber subjected to suddenly applied loads.

Section of factor of safety:-

The section of proper factor of safety used in designing any machine component depends on the following factors.

1) The nature of loading study, live or suddenly applied load
2) homogenity of materials used.
3) The accuracy with which stress in member external forces can be evaluated.
4) The effect of corrosion and wear
5) possible manufacturing, fabraction error
6) The consequences of break down
7) The degree of safety required.

3) The degree of economy desired

Each of the above factor ~~to~~ must be considered and evaluated. The high factor of safety result in highly section & waste of material whereas a low factor of safety result in unnecessary risk of failure.

UNIT-2 (25105160082)

Members subjected to ~~structural~~ structural load.

(structural)

Introduction:

A structural member which is ~~acto~~ acted upon by a system of forces at right angles to its axis is known as beam. And whenever a horizontal beam is loaded with vertical loads some times it bends due to the action of loads. The amount with which the beam bends depends upon the

1) amount and type of the load.

2) length of the beam

3) elasticity of the beam & type of beam.

Shear force & bending moment:

Geometric forms of beams

A beam is defined as the structural member which is used to bear the different loads

→ It can resists vertical force, bending moment, shear force

Types of beams:-

1) Cantilever beam:- 

2) Simply supported beam. 

3) Over hanging beam 

4) Continous beam 

5) Fixed beam 

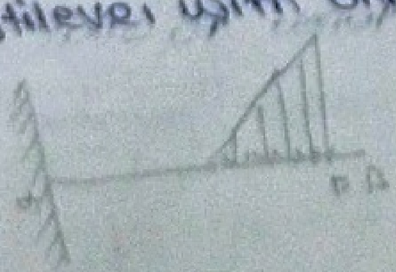
In aircraft structure the most commonly used beams are cantilever & simple supported beam and some times continous

Types of load:-

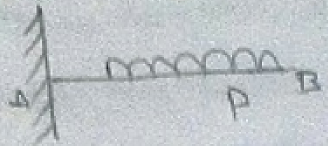
• A beam may be subjected to either in (or) comination of point loads, uniformly varing loads, triangular load, uniformly distributing load.

Cantilever with point load.

Cantilever with uniformly varing load



Cantilever with uniform distributive load.



Shear force:-

A shear force at crossection of a beam is defined as un balanced vertical force to right (or) left of the section

Bending moment

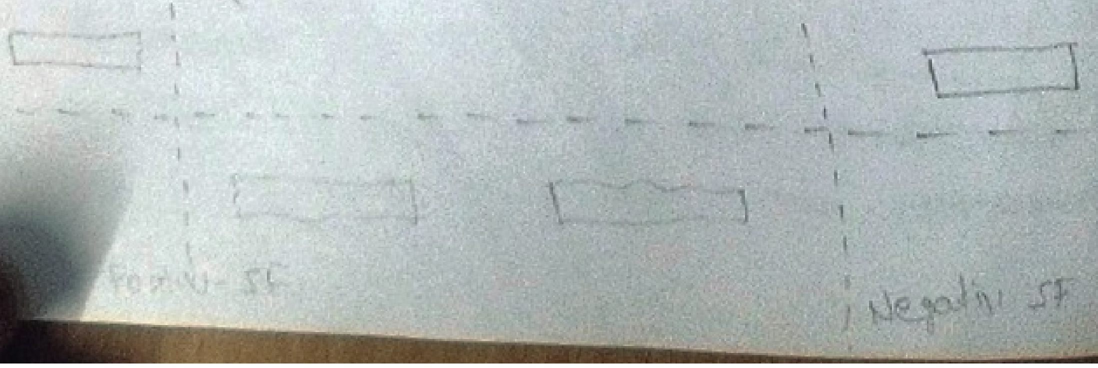
Bending moment at crossection of a beam is defined as algebraic sum of the moments of force to right (or) left of the section.

The diagram representing the variation in shear force. is called a shear force diagram and the diagram representing the variation in load. bending moment is called bending moment

Sign convection of shear force & bending moment:-

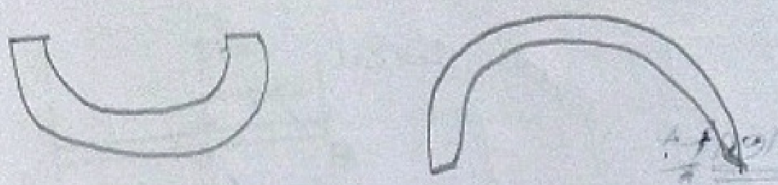
Shear force:-

Shear force at a section is positive. when the left hand portion tends to slide upward (or) right hand poistion tends to slide downward



Bending moment:- (BM)

Because of the load the beam is exprencen sagging postion then it is consider as a positive BM and because of the load the beam expernce hogging postion then it's consider as a Negative BM

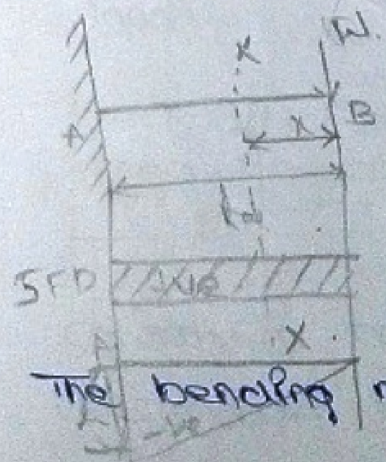


sagging BM (positive)   hogging BM (Negative)

Shear force & bending moment diagram:-

It shows the distrubtion of shear force and bending moment along beam

Cantilever beam with a point load at free end.



The bending moment is

consider a cantilever beam of AB of length 'l' carring point load W at its free end. let us consider a section x-x at a distance of 'x' from the point B (free end)

SF (shear force)

$SF_{xx} = W$

$SF_B = W$

$SF_A = W.$

Bending moment:

$BM_{xx} = -Wx$

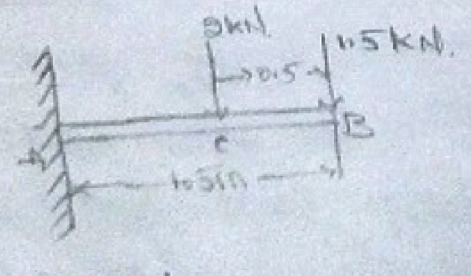
$BM_B = 0.$

$BM_A = -Wl.$

-ve due to hogging.

Draw the shear force and bending moment diagram for a cantilever beam of span length 1.5 meters. carring point four loads as shown in fig.

sol:



$S.F_B = 1.5kN$

$S.F_C = 3.5kN$

$S.F_A = 3.5kN.$

Bending moment:

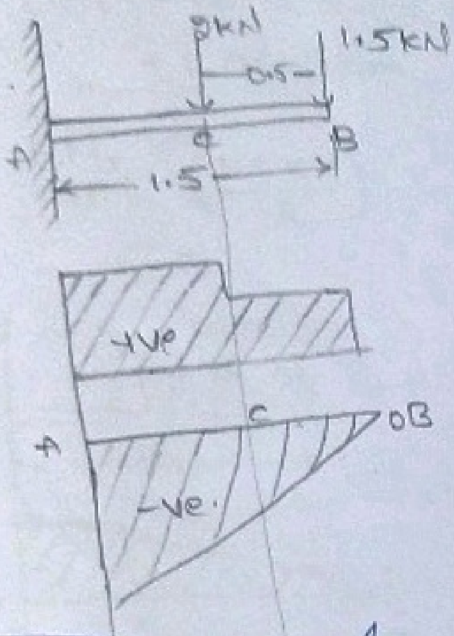
$BM_B = 1.5 \times 0 = 0.$

$BM_C = -(1.5 \times 0.5 + 2 \times 0)$

$= -0.75 \text{ kNm}$

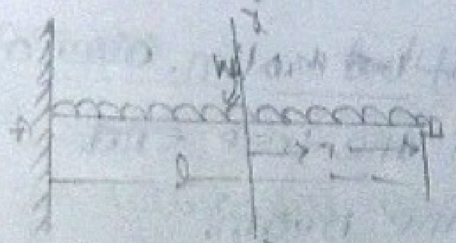
$BM_A = -(1.5 \times 1.5 + 2 \times 1)$

$= -4.25 \text{ kNm.}$



case ii:- formula

A cantilever beam UDL (uniformly distributed load)



consider a cantilever beam of unit length with UDL thought it's length.

let us consider a section x-x at a distance of 'x' from free end.

SF:

$SF_{xx} = Wx$

$SF_B = 0$

$SF_A = Wl$

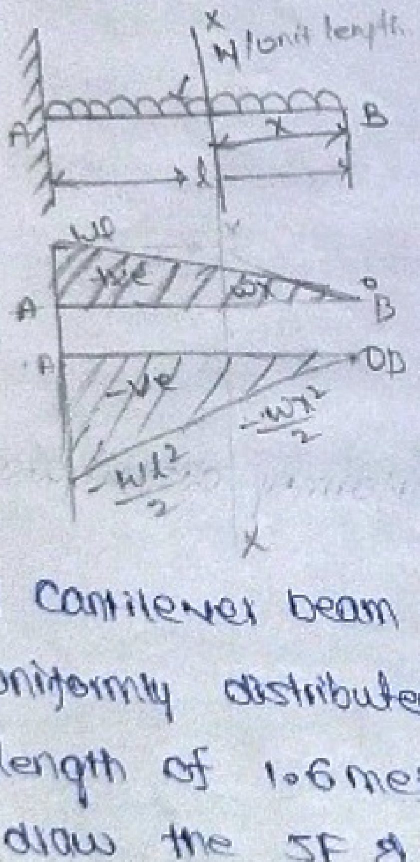
BM:

$BM_B = 0$

$BM_{xx} = (Wx \times x/2)$

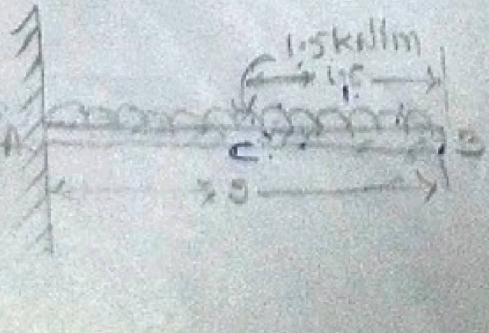
$= -\frac{Wx^2}{2}$

$BM_A = -\frac{Wl^2}{2}$



* A cantilever beam AB 2 meters long carries a uniformly distributed load of 1.5 kN/m. over a length of 1.6 meters from the free end. draw the SF & BM for the beam.

a)



1.5×1.6

$=$

SF

$SF_B = 0$

$SF_A = (1.5 \times \cancel{2}^{1.6} + 4 \times 4 \times 6)$

$= 2.4\,kN \neq 3.$

$SF_C = 1.5 \times 1.6 = 2.4\,kN.$

Bending moment:

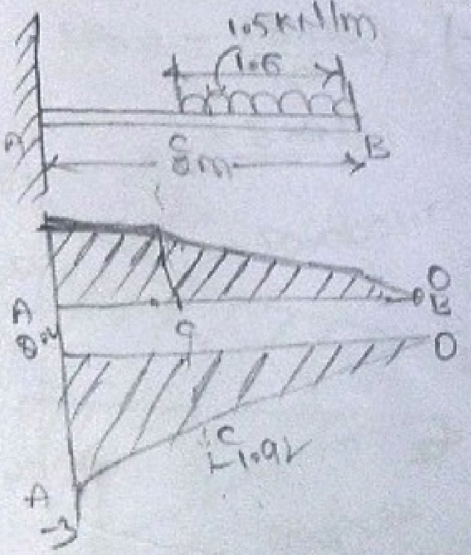
$BM_B = 0.$

$BM_C = -\left(2.4 \times \frac{1.6}{2}\right)$

$= -1.92\,kN/m.$

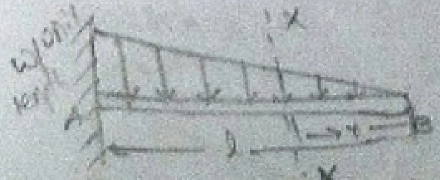
$BM_A = -\left(\frac{1.5 \times 4}{2}\right)$

$= -3.$



case iii :

A cantilever beam with uniformly varing load.
(or) triangular load



consider a cantilever beam of length 'l' carring a uniformly varing load from '0' at free end.

to W/unit length at fixed end. let us consider a section x-x. at a distance 'x' from free end and total load at section x-x

By rule of similar triangle., load at x-x =

$\frac{Wx}{l}$

∴ The total at a small triangular section

$= \frac{1}{2} \times \frac{wx}{2} \times x$

$= \frac{1}{2} \frac{Wx^2}{2l} \times l.$

SF (1)

$SF_B = 0$

$SF_{xx} = \frac{Wx^2}{2l} = (wx)$

$SF_A = \frac{Wl^2}{2l} = \frac{Wl}{2} \text{ ,, } (wl_{,,})$

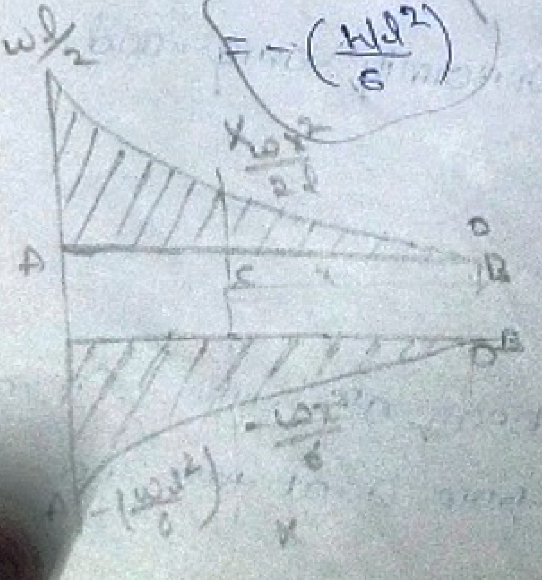
BM

$BM_B = 0$

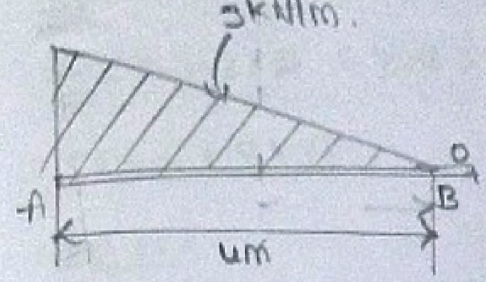
$BM_{xx} = -\left(\frac{Wx^2}{2}\right) \times x/3 = -\frac{Wx^3}{6} \text{ ,,}$

$BM_A = -\frac{Wl^2}{2l} \times l/3.$

$= -\left(\frac{Wl^2}{6}\right)$



A cantilever beam 4 meter long carries a UVL 0 at free end to 3 kN/m at fixed end draw SFD and BMD for the beam.



$UDL = \frac{wl^2}{2}$

$\frac{2 \times 2^2}{2}$

$\frac{2 \times 4}{2} = 4$

$\frac{wl^2}{6} = \frac{3 \times 2^2}{6}$

$4 + 2 = 6$

$SF_B = 0$

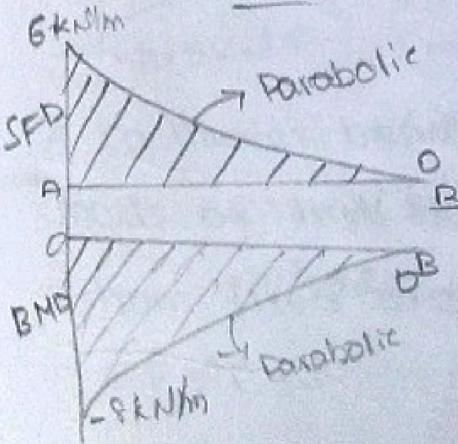
$SF_A = \frac{wl}{2} = \frac{3 \times 4}{2} = \frac{12}{2} = 6\ kN/m.$

BM

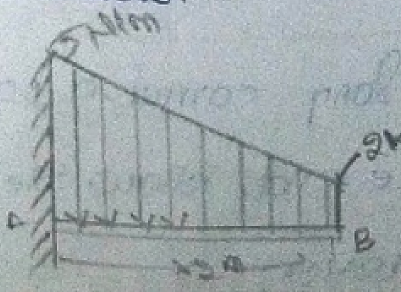
$BM_B = 0$

$BM_A = -\left(\frac{3 \times 16}{6}\right) = -\left(\frac{48}{6}\right)$

$= -8\ kN/m$



A cantilever beam of 2 meter span is subjected to UVL from 2 kN/m to 5 kN/m as shown in below.



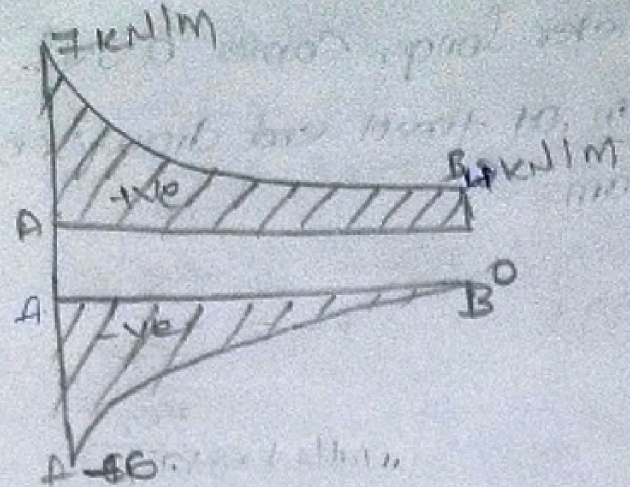
$SF_B = \frac{5+2}{2} \times 2 = 4\ kN/m$

$SF_A = \frac{5+2 \times 2}{2} = \frac{14}{2} = 7\ kN/m$

$BM_B = 0$

$BM_A = -\left(\frac{7 \times 4}{6}\right) = \frac{28}{6}$

$= -4.667$



The load may be assumed to be split up into

① A UDL w_1 of 2KN/m. over the entire span. and

② A gradualy varing load w_2 from 0 to B to 3KN/m at A

$\underline{SF_A}$

The total load to UDL = 2KN/m over the length of 2meter.

The total load = 2×2 = 4KN/m

The total load for UVL = ½ × 2 × 3
= 3KN/m

$SF_B = 0$

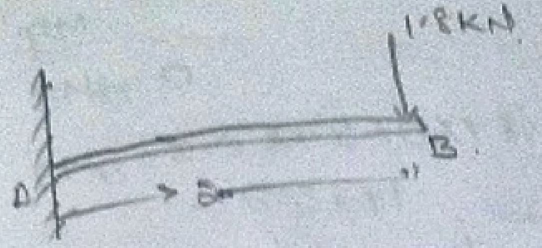
$SF_A =$

$$UDL = \frac{wl^2}{2} = \frac{2\times4}{2} \quad 8/2 = 4$$

$$UVL = \frac{wl^2}{6} = \frac{3\times2^2}{6} = 4/2 = 2$$

UDL + UVL = 4 + 2 = -6.

A cantilever beam 2meter long carries a point load of 1.8KN. at is free end. draw the S.F & B.M for the cantilever.



SF.

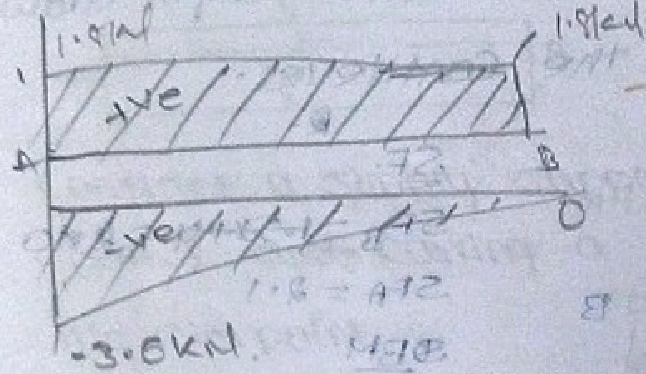
$SF_A = 1.8 \times 2 = 3.6 = 1.8\ KN$

$SF_B = 1.8KN$

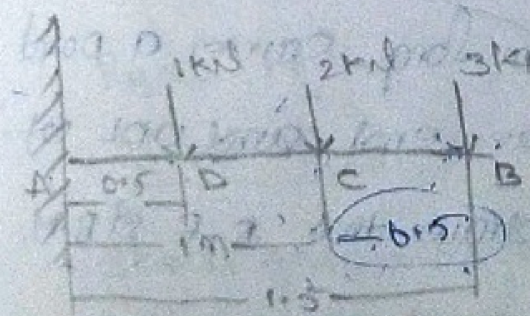
BM:-

$BM_A = -(1.8 \times 2)$, $B.M = 0$

$= -3.6\ KN$



A cantilever beam 1.5 meter long carries point loads of 1KN, 2KN & 3KN at 0.5m, 1m, & 1.5 from fixed end respectively. draw the S.F & B.M



SF

$SF_A = 6KN$

$SF_B = 3KN$

$SF_C = 5KN$

$SF_D = 6KN$

BM

$BM_B = 0$

$BM_C = 3 \times 0.5$

$= 1.5KN$

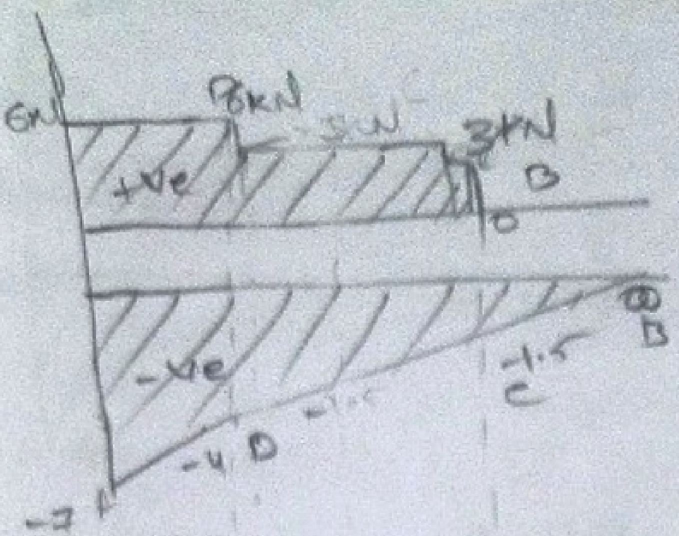
$BM_D = (3 \times 1 + 2 \times 0.5)$

$= 4KN$

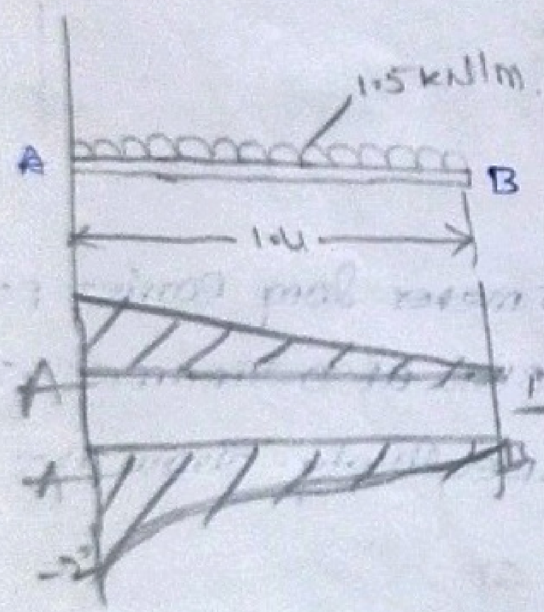
$BM_A =$

$-(3 \times 1.5 + 2 \times 1 + 1 \times 0.5)$

$= 7KN$



A Cantilever beam 1.4 m length carries a UDL of 1.5 kN/m over its entire length draw S.F and B.M for the Cantilever.



SF.

$SF_B = \cancel{1.5 \times 1.4} = 2.10$

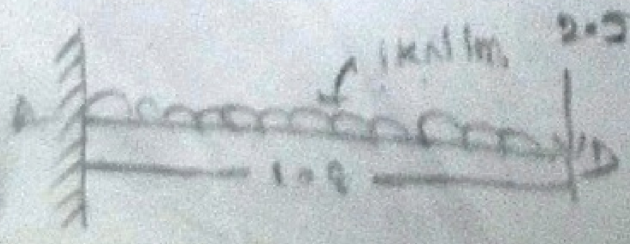
$SF_A = 2.1$

BM

$BM_B = 0$

$BM_A = 1.5 \times 1.4$
$= -2.1$

A cantilever beam of 1.8 m long carries a point load of 2.5 kN at free end and UDL of 1 kN/m from A to B draw the S.F & B.M.



SF

$SF_B = 2.5$

$SF_A = 2.5 + 1 \times 1.8$
$= 4.3$

BM

BM at B

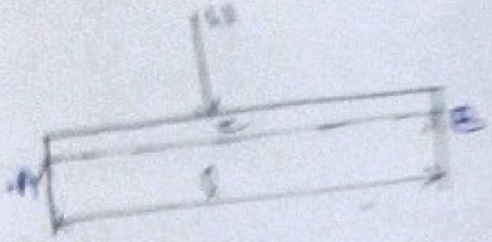
$BM_B =$ [illegible]

$= 6.3\,kN$

Simply supported beam:

case 1:

(1) Simply supported beam with a point load at it's mid point



Consider a simply supported beam AB of span 'l' and carrying a point load W at it's mid point.

System of forces

$R_A + R_B + (-W) = 0$

$R_A + R_B - W = 0$

$\boxed{R_A + R_B = W}$

$R_B \times l - W\, l/2 = 0$ [illegible]

$\boxed{R_B = W/2}$ $\boxed{R_A = W/2}$

Shear force at B

[illegible]

BM

$BM_B = 0$

$BM_A = 8.5 \times 1.8 + (1 \times 1.8)$

$= 6.3\ kN$

Simply supported beam:

case (i):

(1) Simply supported beam with a point load at it's mid point

consider a simply supported beam 'AB' of span 'l' and carring a point load 'W' at it's mid point.

System of forces

$R_A + R_B + (-W) = 0$

$R_A + R_B - W = 0$

$\boxed{R_A + R_B = W}$

$R_B \times l - W l/2 = 0$ $\quad$ $R_A \times l - W l/2 = 0$

$\boxed{R_B = W/2}$ $\quad$ $\boxed{R_A = W/2}$

shear force at B

$SF_B = -W/2$ $\quad$ $SF_A = -W/2 + W + W/2 = 0$

$SF_C = -W/2 + W = W/2$

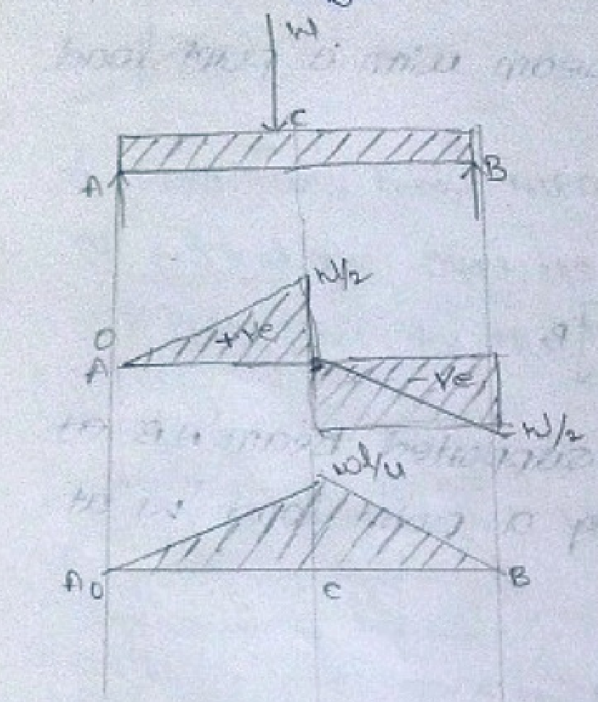
Bending moment:

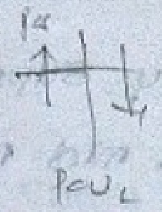
$BM_B = W/2 \times 0 = 0.$

$BM_C = -W/2 \times l/2 = -Wl/4.$

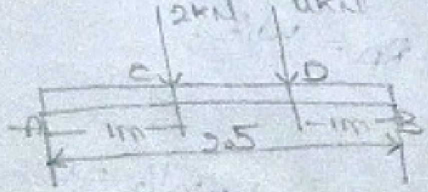
$BM_A = W/2 \times l - Wl \times l/2$

$= 0.$





A simply supported beam AB of span 2.5m. is carring two point load. as shown in fig draw the shear force and bending moment diagram for the beam.



Shear forces:

$SF_A =$

$R_B \times 2.5 - (4 \times 1.5) - (2 \times 1 = 0)$

$R_B \times 2.5 - 4 \times 1.5 - 2 \times 1 = 0$

$R_B = 3.2$

$\boxed{R_A + R_B = 6}$

$R_A + 3.2 = 6$

$\boxed{R_A = 2.8}$

$SF_C = 2.8$

$= 2 + 4 - 3.2$

$6 - 3.2 = 2.8$

$SF_D =$

$SF_D = 4 - 3.2$

$= 0.8$

$SF_D = -3.2 + 4$

$= 0.8$

$SF_C = -3.2 + 6$

$= 2.8$

Bending moment:

$BM_B = 0 \quad -3.2 \times 0 = 0$

$BM_A = 0 \quad 2.8 \times 0 + 2 \times 1 + 4 \times 1.5 - 3.2 \times 2.5 = 0$

$BM_D = 4 \times 0 - 3.2 \times 1$

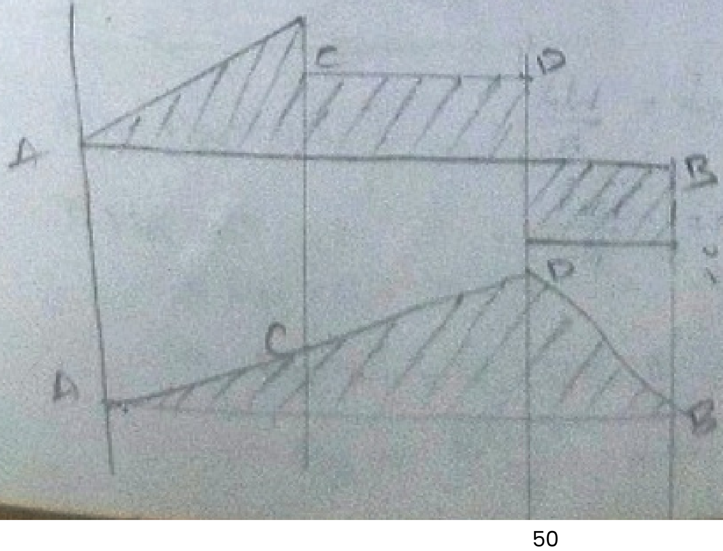
$= -3.2$

$BM_C = 4 \times 0.5 - 3.2 \times 1.5 = -2.8$

$BM_B = 0, \; BM_D = -3.2 \times 1 = -3.2$ (but '+' because of sagging)

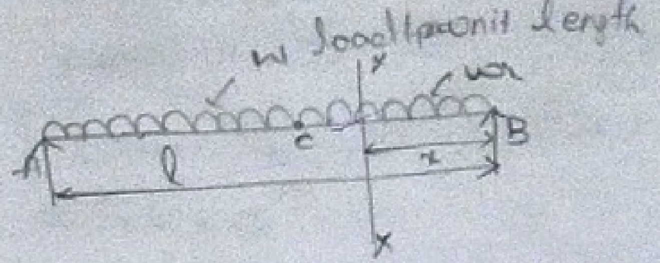
$BM_C = (-3.2 \times 1.5) + (4 \times 0.5) = -2.8$ (but '+' because of sagging)

$BM_A = (-3.2 \times 2.5) + (4 \times 1.5) + (2 \times 1) = 0$



Case ii

A simply supported bean UDL



consider a simply supported beam of length 'l' carring UDL of 'w'/unit length throughout

Now,

$R_A + R_B = wl$.

$-R_B \times l + \frac{wl^2}{2} = 0$.

$\boxed{R_B = wl/2}$

$R_A + wl/2 = wl$.

$-R_A \times l + wl/2 = wl$

$\boxed{R_A = wl/2}$

let us consider a small section 'x-x' at a distance 'x' from B. Now.

$SF_{xx} = -wl/2 + wx \cdot SF_C = 0$

$SF_B = -\frac{wl}{2}$.

$SF_A = -wl/2 + wl = \frac{wl}{2}$.

just before $A = -\frac{wl}{2}$.

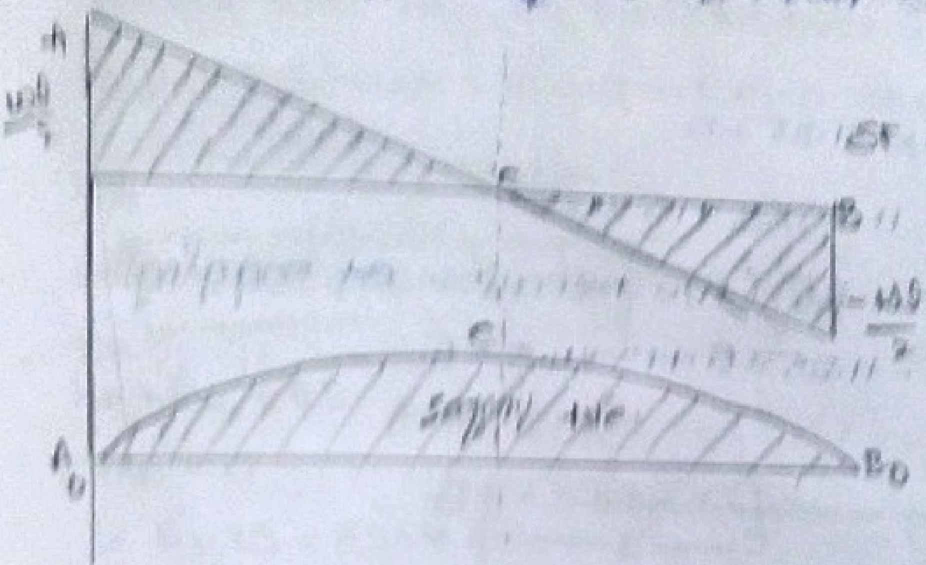
Bending moment:

$BM_B = 0$

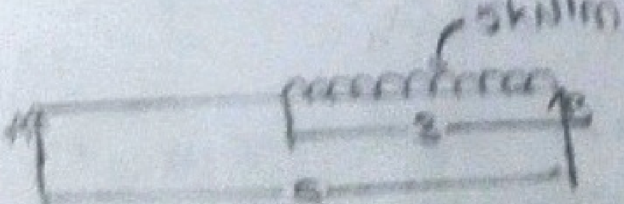
$BM_E = -\frac{wl}{2}(l/2) + \frac{w(l/2)^2}{2}$ $[\because x = l/2]$

$= -\frac{wl^2}{4} + \frac{wl^2}{8} = \frac{-2wl^2 + wl^2}{8} = -\frac{wl^2}{8}$

$BM_A = -\frac{wl^2}{2} + \frac{wl^2}{2} = 0$ $[\because x = l]$



Q) A simply supported beam of 6m long is carring a uniform distributed load of 5kN/m over a length of 3m from right end. draw SF & BM of beam and also BMmax.



$R_A + R_B = 15$

$\Sigma M_A = 0$

$= R_B \times 6 - 15 \times \overset{4.5}{3} = 0$

$= R_B \times 6 - 15 \times 4.5 = 0$

$R_B = \frac{15 \times 4.5}{6} = 11.25$

$R_B = 11.25$

$R_A = 15 - 11.25$ KN.

$= 3.75$ kN

SF

$SF_B = -11.25$

$SF_C = -11.25 + 15 = +3.75$

$SF_A = 0 - 11.25 + 15 - 3.75 = 0$

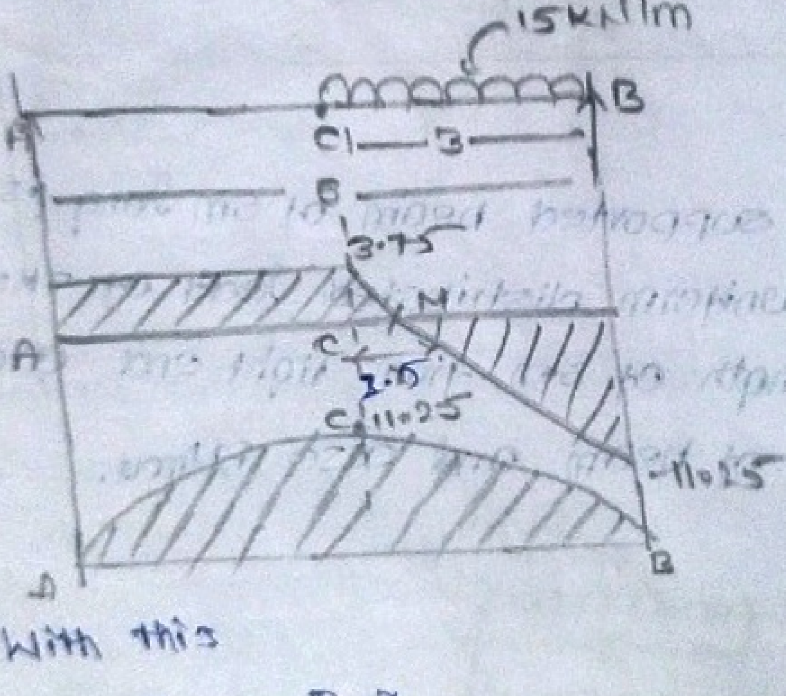
BM

$BM_B = 0 \times -11.25 = 0$.

$BM_C = -11.25 \times 3 + 15 \times 1.5$

$= -11.25$ (+ve because of sagging)

$BM_A = -11.25 \times 6 + 15 \times 4.5 = 0$.



With this

$$\frac{x}{3.75} = \frac{3-x}{11.25}$$

$11.25x = 3.75(3-x)$

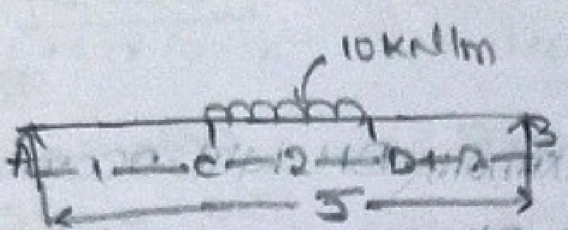
$11.25x = 11.25 - 3.75x$ $\quad$ BM = 11.06

$15x = 11.25$

$x = \frac{11.25}{15}$ $\quad \boxed{x = 0.75}$

We know that maximum BM will occur at 'M' where shear force changes sign at 'x' be the distance b/w 'c' & 'M' from the geometry of the fig b/w 'c' & 'B' we have the above calculation.

Q) A simply supported beam of 5m long is loaded with UDL of 10KN/m over a length of 2m as shown in fig. Draw SFD & BMD.



$R_A + R_B = 20$

$\Sigma M_A = 0$

$= R_B \times 5 - 20 \times 2 = 0.$

$R_B \times 5 - 40 = 0$

$R_B = \frac{40}{5} = 8$

$R_B = 8$

$R_A + 8 = 20$

$\boxed{R_A = 12}$

SF

$SF_B = -8$ $\quad SF_C = -8 + 20 \quad SF_A = -8 + 20 - 12 = 0.$

$SF_D = -8$ $\quad SF_C = 12.$

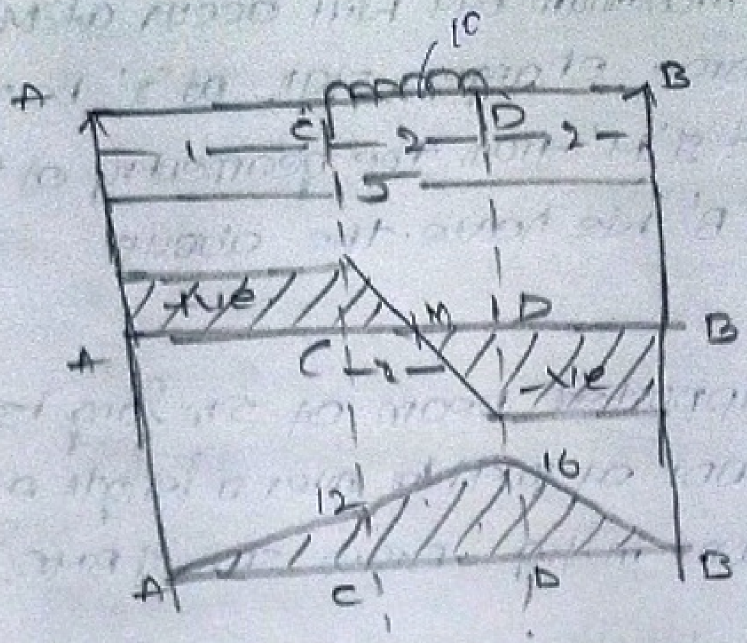
BM

$BM_B = 0, \; BM_D = -8 \times 2 = -16.$

$BM_C = -8 \times 4 + (10 \times 1) =$ (+ve sagging)

$= -32 + 10 = -12$ (+ve sagging)

$BM_A = -8 \times 5 + 20 \times 2 = 0.$



We known that maximum BM will occur at 'M' where shear force changes sign at 'x' be the distance b/w 'C' & 'M'

$$\frac{x}{12} = \frac{2-x}{8}$$

$$8x = 24 - 12x$$

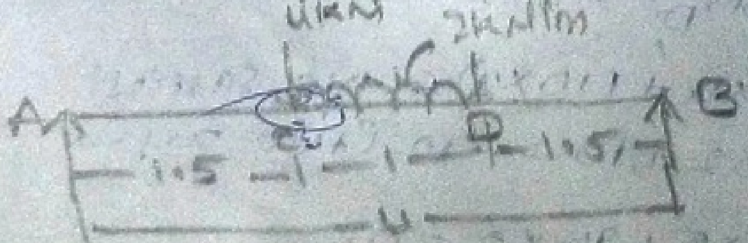
$$20x = 24$$

$$x = \frac{24}{20} = 1.2$$

$\frac{1}{2} \times 4 \times 12 = 8$

$$\boxed{BM_{max} = 19.2\,kN}$$

Q4 A simply supported beam of 4m span is loaded as shown in fig.



$R_A + R_B = 6$

$\Sigma M_A = 0$

$R_B \times 4 - 2\times 2 - 4\times 1.5 = 0$

$R_B \times 4 - 4 - 6 = 0$

$R_B = \frac{10}{4} = 2.5$

$R_A + 2.5 = 6$

$R_A = 6 - 2.5$

$\boxed{R_A = 3.5}$

SF

$SF_B = -2.5$ $\quad SF_D = -2.5 + 2 + 4 - 3.5 = 0$

$SF_D = -2.5$

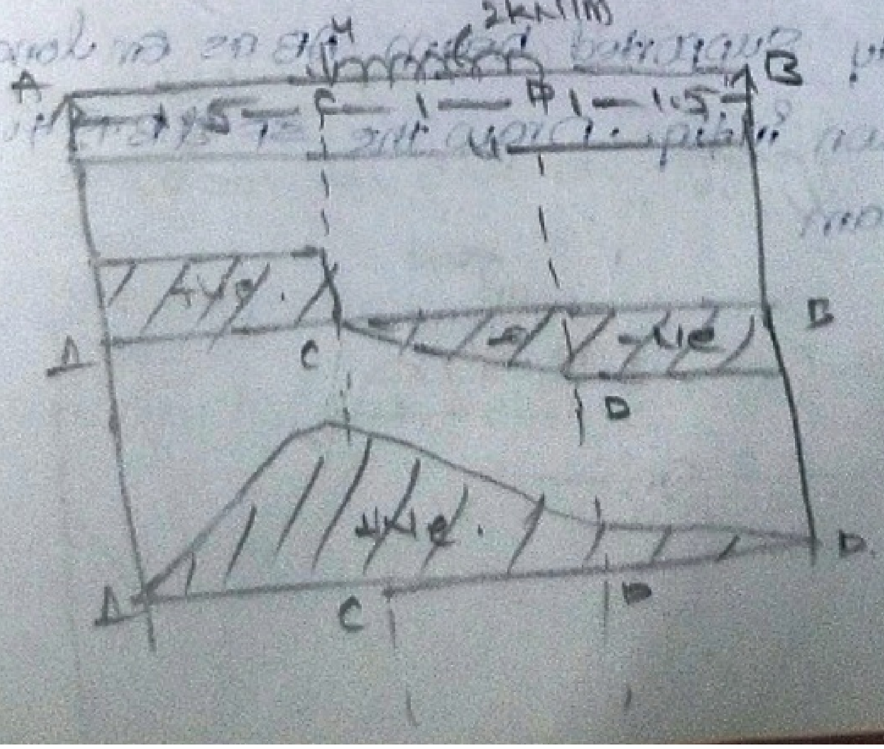
$SF_C = -2.5 + 2 + 4 = -2.5 + 6 = 3.5$

BM

$BM_B = 0$

$BM_C = (-2.5 \times 2.5) + (2 \times 0.5) = 5.25$

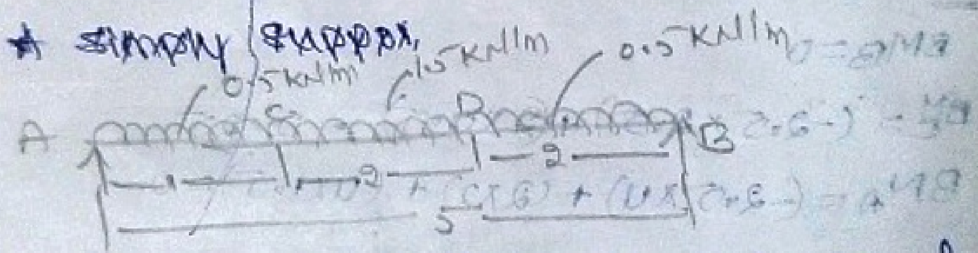
$BN_A = (-2.5 \times 4) + (2 \times 2) + (4 \times 1.5) = 0.$



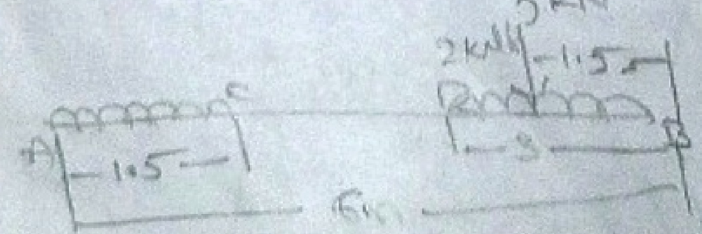
1) A simply supported beam of 8 meter span carries two load of 5kN each at 1m & 2m from left hand support. draw the shear force and bending moment diagram for the beam

2) A simply supported beam of span L=5m carries a uniformly distributed load of 3.6kN/m over a length of 2m from the left end. A. Draw the S.F & B.M diagram

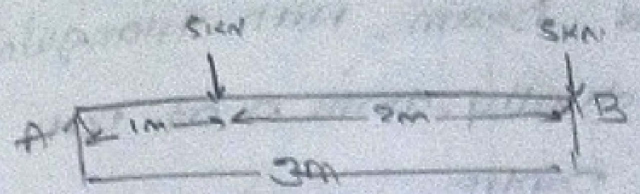
3) Draw the S.F & B.M diagram for a simply supported beam loaded as shown in fig. find the position & maximum bending moment that will occure in the Beam



4) A simply supported beam AB as 6m long as shown in fig. Draw the SF & B.M for the beam



1A



SF

$R_A + R_B = 10kN$

$SF_B = -6.667 + 5$

$= -1.667$

$SF_A = -6.667 + 5 + 5 - 13.333$

$= 0$

$SF_C = 5kN$

SF & B.M

$1.5x^2$

3.0

$SM_A = R_B \times 3 - 5\times3 - 5\times1$

$= R_B \times 3 - 15 - 5 = 0$

$R_B = 20/3 = 6.667$

$R_A + 6.66 = 20$

$R_A = 20 - 6.667$

$= 13.333$

BM

$BM_B = 0$

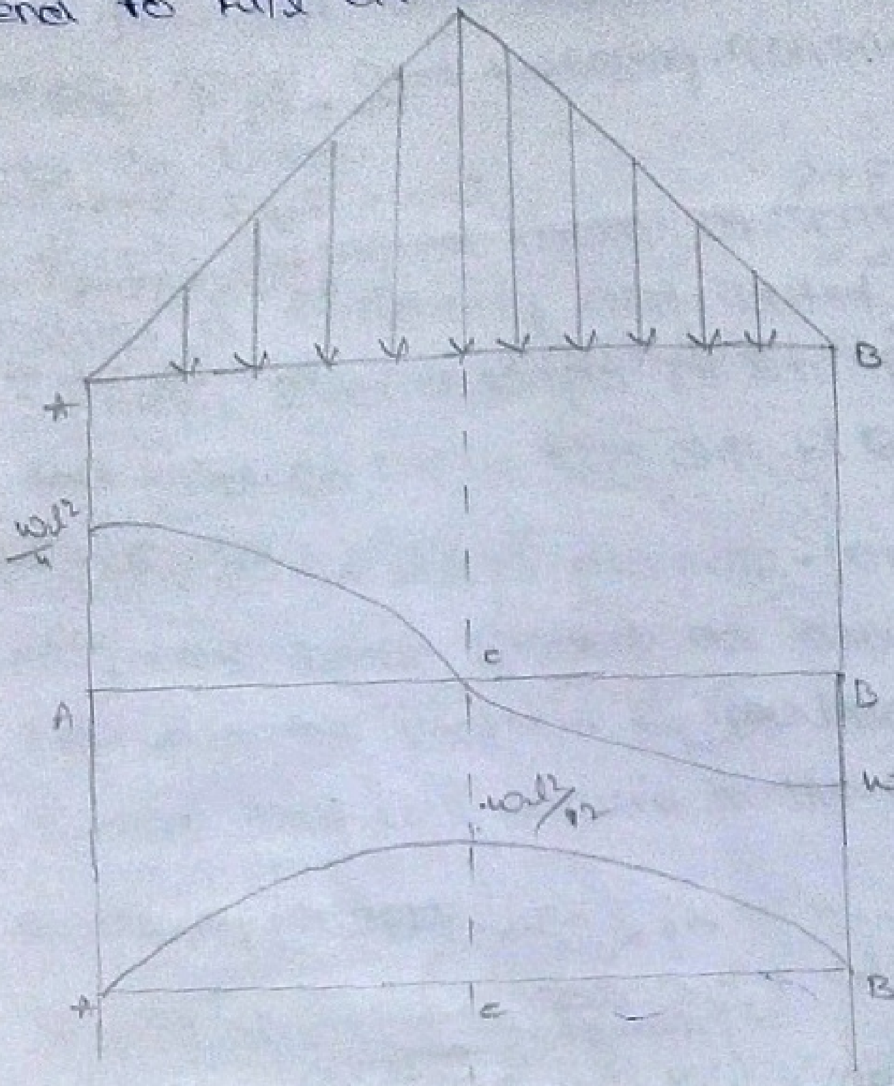
$BM_C = -1.667 \times 3$

$= -5.001$ (+ve due to Sagging)

$BM_A = -1.667 \times 3 * 5 \times 1$

$= 0.001$ (+ve due to Sagging)

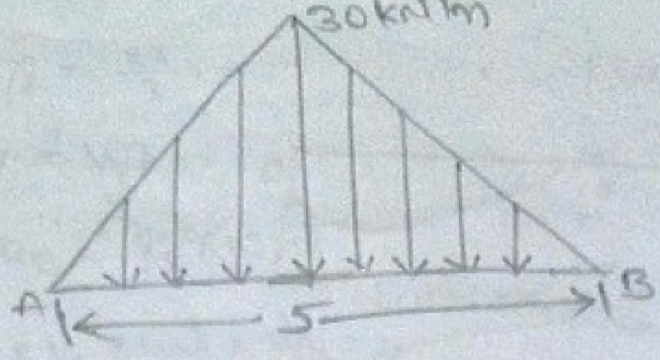
Simple supported beam with triangular load varing gradually from 0 at both end to W/l at centre.



$$BMA = -\frac{wl^2}{4} + \frac{wl}{4}\left(\frac{l}{2} + \frac{l}{6}\right) + \frac{wl}{4}\left(\frac{2l}{6}\right)$$

$$= -\frac{wl^2}{4} + \frac{2wl}{12} + \frac{2wl^2}{24}$$

Q) A simply supported beam of span 6m carries a triangular load of 30kN/m at centre draw SF & B.M



Sol: $R_A + R_B = 75$

$= -R_B \times 5 + 37.5\left(2.5 + \frac{2.5}{3}\right) + 37.5\left(\frac{2}{3} \times 2.5\right) = 0.$

$-R_B \times 5 + 125 + 62.5 = 0$

$-R_B = \frac{187.5}{5}$

$R_B = 37.5$

SF

$SF_B = -37.5$

$SF_C = -37.5 + 37.5 = 0.$

$SF_A = -37.5 + 37.5 + 37.5 = 37.5$

(right of A)

$SF_A = 0.$

BM:

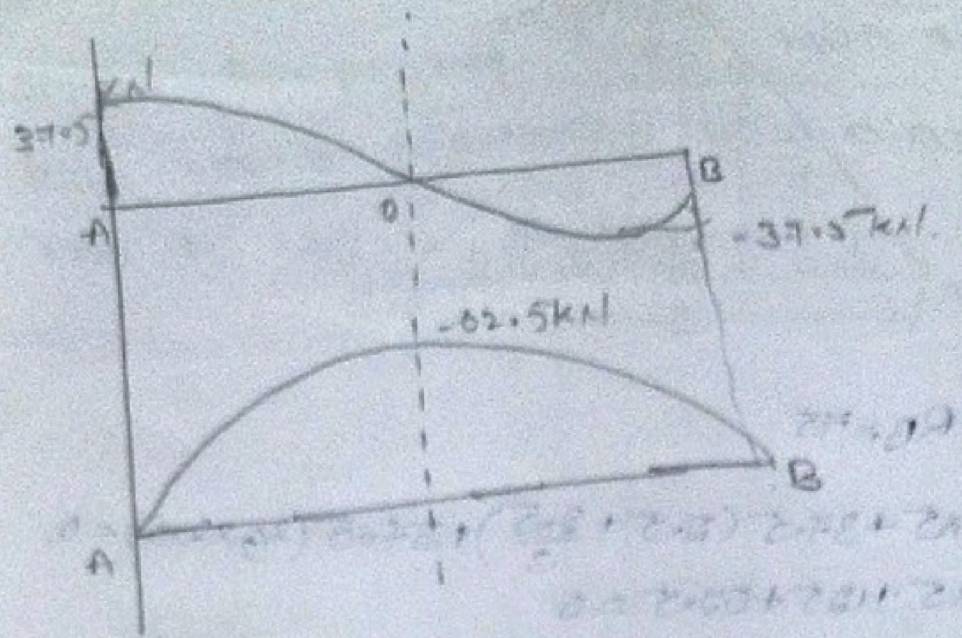
$BM_B = 0$

$BM_C = -37.5 \times 2.5 + 37.5 \times 2.5/3$

$= -93.75 + 31.25$

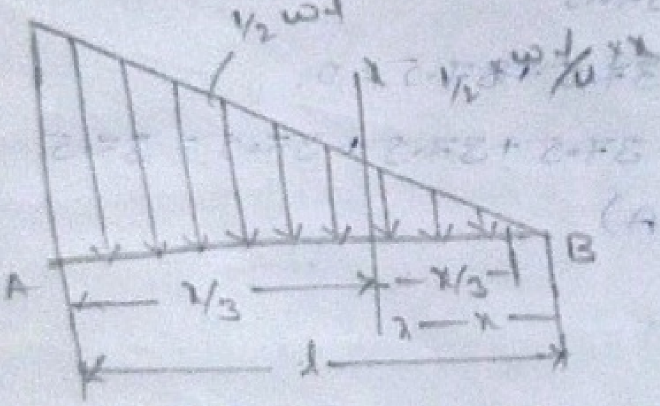
$= -62.5$ kN/m.

$BM_A = 0.$



Case4:-

simply supported beam with U.V.L from zero at one end to w/l.



consider a simply supported beam of 'AB' of length 'l' and carrying gradually varing load '0' at one end and w/l at other end. since the load is varying gradually from zero at one end to w/l at other

∴ both the reaction at a and b will be

By rule of similar triangle Δ

$$\omega/l = \frac{xx}{x}$$

$$xx = \frac{\omega x}{l}$$

$$R_A + R_B = \frac{1}{2} \times \omega \times l$$

$$-R_A \times l \times \frac{1}{2} \times \omega l \times l/3 = 0$$

$$R_B l = \frac{\omega l^2}{6}$$

$$\boxed{R_B = \frac{\omega l}{6}}$$

$$R_A = \frac{\omega l}{2} - \frac{\omega l}{6}$$

$$\boxed{R_A = \frac{\omega l}{3}}$$

SF

$$SF_{xx} = -\frac{\omega l}{6} + \frac{\omega x^2}{2l}$$

$$SF_B = 0$$

$$SF_{A1} = -\frac{\omega l}{6}$$

$$SF_A = -\frac{\omega l}{6} + \frac{\omega l^2}{2l}$$

$$SF_A \text{ (right of A)} = \frac{\omega l}{3}$$

$$SF_A = 0.$$

BM

$$BM_{xx} = -\frac{\omega l}{6} \times x + \frac{\omega x^2}{2l} \times x/3$$

$$= -\frac{\omega l x}{6} + \frac{\omega x^3}{6l}$$

$$BM_B = 0$$

$$BM_A = 0 = \frac{\omega l^2}{6} - \frac{\omega l^3}{6l} = 0.$$

BM_{max} will occur where $SF = 0$. let us to find the distance when $SF = 0$.

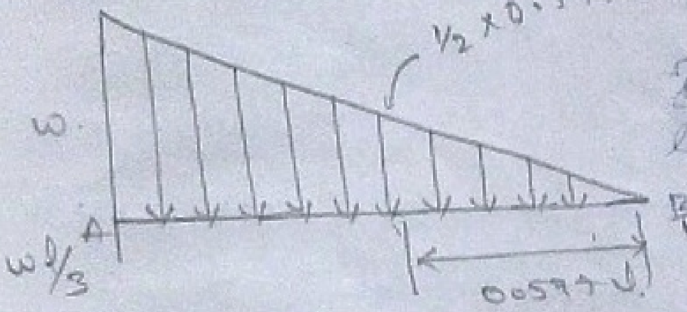
$$\therefore -\frac{wl}{6} + \frac{wx^2}{2l} = 0$$

$$= \frac{wl}{6} = \frac{wx^2}{2l}$$

$$\Rightarrow x^2 = \frac{2l^2}{6} = l^2/3$$

$$x = l/\sqrt{3} = 0.577(l)$$

i) simply supported beam with U.V.L from.



$$BMax = -\frac{wl}{6} \times 0.577 w \times 0.577\, l \times \frac{0.577\, l}{3}$$

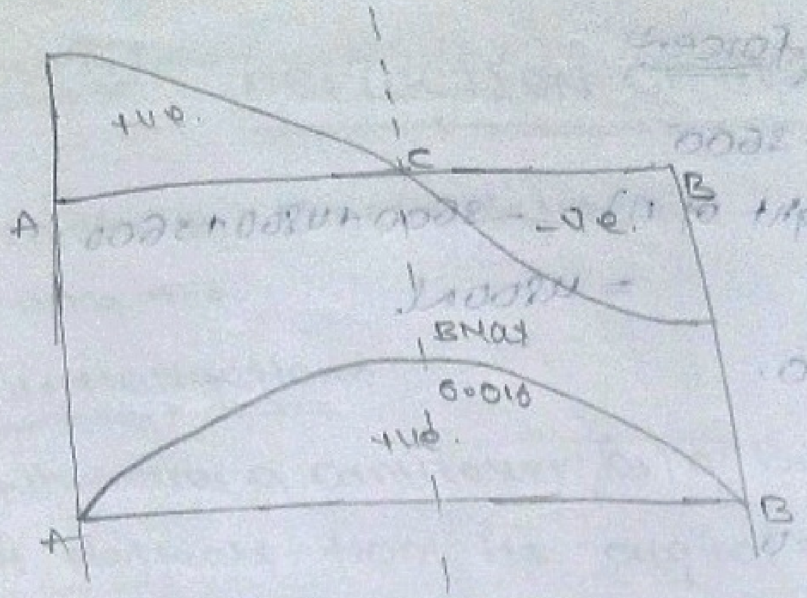
$$= -0.096 \cdot \frac{wl^2}{6} + 0.032\, wl^2$$

$$= -0.016 wl^2 + 0.32\, wl^2$$

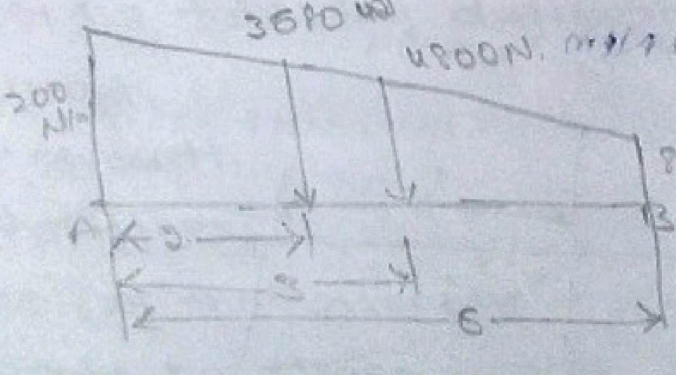
$$BM_{max} = 0.016\, wl^2$$

$$w/l = \frac{xx}{x} \qquad w/l = \frac{xx}{0.577\, l}$$

$$xx = \frac{0.577\, wl}{l} \qquad xx = 0.577\, w.$$



14* the intensity of loading on a simply supported beam of "6m" increases gradually 800 N/m at one end to 200 N/m at other as the intensity of loading find the position of amount of maximum loading m also draw the SF & BM diagram.



$R_A + R_B = 3600 + 4800$

$= 8400$

$-R_B \times 6 + (4800 \times 3) + (3600 \times 2) = 0$

$R_B = 3600\ N$, $R_A = 8400 - 3600\ N$
$= 4800\ N$.

Shear force

$SF_B = -3600$

SF_A (right of A) $= -3600 + 4800 + 3600$

$= 4800 N.$

$SF_A = 0.$

BM

$BM_B = 0$

$BM_A = 0$

* to find 'x'

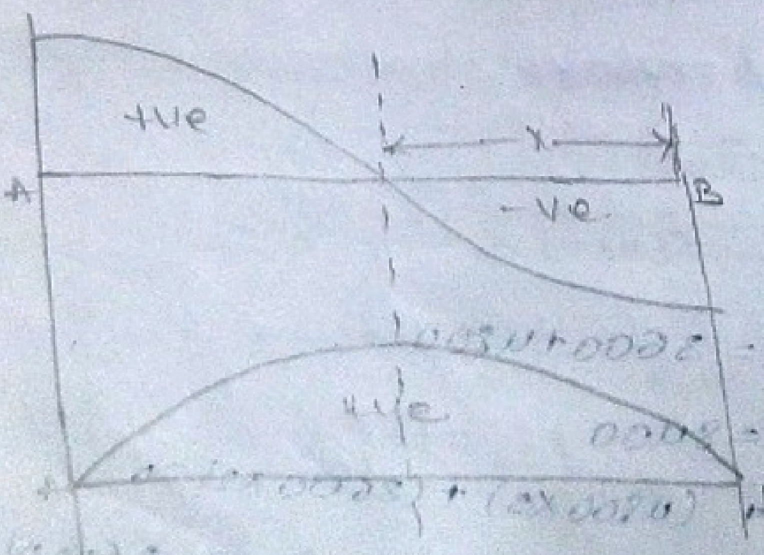
$SF_{xx} = 0$

$x = \frac{-8 \pm \sqrt{8^2 + (4 \times 36)}}{2}.$

$x = 3.2 m$

Max BM

$= -6332 N{\cdot}m$



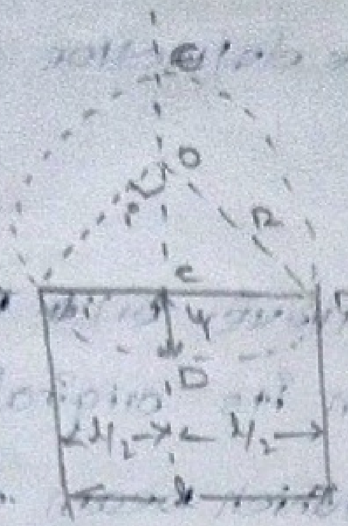
UNIT-3 DEFLECTION OF BEAMS:-

* cantilever, simple deflection
* using the.

* Introduction:-

* whenever a cantilever (or) a beam is loaded it deflects from its original position.
* The amount by which beam deflects depends upon.
  - → It's cross section
  - → & Bending moment.
* It's modern design following are the strength, stiffness two design criteria for the following deflection cantilever. beam.
  1) strength
  2) stiffness.
* as per the strength criterion the beam should be strong enough to resist the bending stresses & shear stresses
* as per the stiffness criteria the beam should be stiff enough not to deflect more than the Permissable limit under the action of load.

Curvature of a bending beam:-



* consider a beam "AB" subjected to a bending moment. as a result of loading let the beam deflect from "ACB" to "ADB" into a circular arc, as shown fig

* let "l" length of the beam "AB"

  "M" Bending moment

  "R" Radius of curvature of the bend beam

  "I" - moment of inertia of the beam section

  "E" - modulus of elasticity of the beam.

  "y" - deflection of the beam i.e (CD)

  "i" - slope of the beam (i.e angle whole tangent at "A" make with "AB")

* from the geomentry of the circle we know that :

$AC \times CB = EC \times CD$

(or)

$l/2 \times l/2 = EC \times y$

$y \times EC = l^2/4$

$(2R-y)\, y = l^2/4$

$2Ry - y^2 = l^2/4$ [$\because$ neglecting y^2]

$2Ry = R^2/4$ ($\because y \approx 0$)

$$\boxed{y = l^2/8R} \quad (1)$$

* But we know that for a loaded beam

$\frac{M}{I} = E/R$

$$\Rightarrow \boxed{R = \frac{EI}{M}}$$

Sub 'R' in (1)

$$\boxed{y = \frac{Ml^2}{8EI}}$$

* from the geomentry of the fig we found the slope of the beam "i" at 'A' (or) "B" is also equal to angle "$\angle AOC$"

$\sin i = \frac{AC}{OA} = \frac{l/2}{R} = l/2R$

* since the angle "i" is very small

$\therefore$ sin i angle taken equal to,

$i = l/2R$ (radian) ("i" is very small)

* Substitue the value of R in eq ②

$i = l/2R$

sub "R" in it

$$i = \frac{l}{2 \times \frac{EI}{M}}$$

$$i = \frac{Ml}{2EI} \text{ radius}$$

* **Note**~

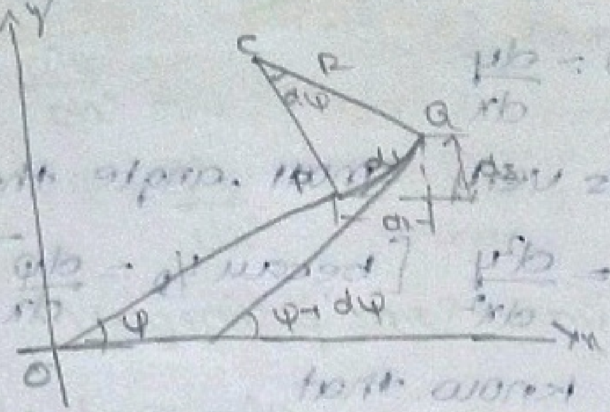
1) The above eq^n for deflection "y" a slope "i" have been divided from the bending moment.

2) In actual practice the beam bent in the arc of a circle only in few cases. a little consideration will show that beam will bend into an arc of a circle only if

1) The beam is of uniform section.

2) The beam is subjected to constant moment throughout its length or the beam is of uniform strength.

* **Relation b/w slope, deflection, radius of curvature**



* Now, consider small portion "PQ" of beam. bent into an arc as shown in fig above.

* let ds = length of the beam PQ

  R = radius of arc, into which the beam has been bent

  c = centre of the arc.

  φ = angle, which the tangent makes at P makes at xx axis

  $\varphi + d\varphi$ = angle, which the tangent makes with x-x axis

From the geomentry of the fig.

$\angle PCQ = d\varphi$

$ds = R \cdot d\varphi$

$R = \frac{ds}{d\varphi}$ ($\because$ consider $ds = dx$),

$= \boxed{R = \frac{dx}{d\varphi}}$, $\boxed{1/R = \frac{d\varphi}{dx}}$

W.K.T

if "x" & "y" be the co-ordinate of point 'p' then $\tan\varphi = \frac{dy}{dx}$

$$\tan\varphi = \frac{dy}{dx}$$

Since, φ is very small angle theretom.

$$\frac{d\varphi}{dx} = \frac{d^2y}{dx^2} \quad \left[\text{becau } 1/R = \frac{d\varphi}{dx}\right]$$

We also know that

$$\frac{M}{I} = E/R$$

$$\boxed{M = EI \times \frac{d^2y}{dx^2}} \quad \left(\text{subsutiting the value of } 1/R\right)$$

* **Method for slop & deflection:**

* there are many methods to find out the slope and deflection of a beam i.e. loaded the following are 2 method are imp for the subject point of view.

they are of,

i) Double integration method

ii) Macaulay's method.

i) **Double integration method for slop & deflection:**

* here

$$\frac{M}{I} = E/R \qquad M = \frac{EI}{R}$$

$1/R = \frac{d^2y}{dx^2}$

$M = EI \times \frac{d^2y}{dx^2}$

Now integrating the above eq

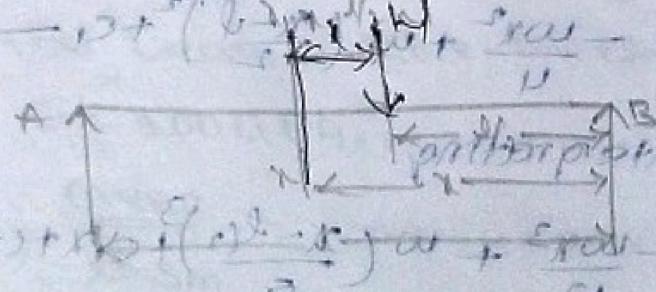
$\int M = \int EI \frac{d^2y}{dx^2}$

$\int M = EI \frac{dy}{dx}$ → gives slope

$\iint M = \int EI \frac{dy}{dx}$

$\iint M = EIy$ → gives deflection

S.S beam with a central point load:



$R_A + R_B = W$

$-R_B \times l + W \times l/2 = 0$

$R_B = W/2$

$R_A = W/2$

Now, the moment about the section x-x is M_{xx} = moment about xx section.

$= -R_B \times x + w(x - l/2)$

but from moment eqn w.k.T

$$M = EI \cdot \frac{d^2y}{dx^2}$$

i.e $EI \cdot \frac{d^2y}{dx^2} = -R_B x + w(x - l/2)$

$$EI \cdot \frac{d^2y}{dx^2} = -\frac{wx}{2} + wx - \frac{wl}{2}$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{wx}{2} - \frac{wl}{2}$$

$$EI \frac{d^2y}{dx^2} = w/2 \, (x - l)$$

Now integrate the above eqn

we get

$$EI \cdot \frac{dy}{dx} = -\frac{wx^2}{4} - \frac{wxl}{2} + C_1$$

$$EI \cdot \frac{dy}{dx} = -\frac{wx^2}{4} + w \cdot \left(\frac{x - l}{2}\right)^2 + C_1 \quad \text{--- (1)}$$

again integrating

$$EIy = -\frac{wx^3}{12} + w\left(\frac{x - l/2}{6}\right)^3 + C_1x + C_2 \quad \text{--- (2)}$$

apply the boundary condition

i) at 'B' $\qquad$ ii) at "A"

$x = 0$ $\qquad$ $x = l$

$y = 0$ $\qquad$ $y = 0$

$l = 0$

apply these in eqn 1 & (2)

(1) → (i) at "B" $\qquad$ (2) (i) at "B"

$C_1 = 0$ $\qquad$ $C_2 = 0$.

① $\Rightarrow$ i, ii) at "A"

$$EI(0) = -\frac{wl^2}{4} + w\frac{(l+l)^2}{2} + C_1.$$

$$C_1 = \frac{wl^2}{4}.$$

② $\Rightarrow$ iii) at A

$$EI \cdot 0 = -\frac{wl^3}{12} + \frac{wl^3}{48} + C_1 l + 0 = 0.$$

$$C_1 l = \frac{wl^3}{12} - \frac{wl^3}{48}$$

$$C_1 l = \frac{3wl^3}{48}$$

$$\boxed{C_1 = \frac{3wl^2}{48}}$$

① A simply supported beam of span "3m" is subjected to the central load of. 10kN find the max slope and deflection of the beam. take $i = 12 \times 10^5 mm^4$ &

$E = 200 GPA$

Sol: Given

$l = 3m$

$= 3 \times 1000$

$= 3 \times 10^3 mm$

$W = 10kN$

$= 10 \times 10^3 N.$

$i = 12 \times 10^5 mm^4$

$E = 200 GPa = 200 \times 10^9 PA = 200 \times 10^3 N/mm^2$

$$\text{Max slop} = \frac{Wl^2}{16EI} = \frac{0.0023}{(\text{radius})} = 2.23 \times 10^{-3}\,mm$$

$$\text{deflection} = \frac{Wl^3}{48EI} = 2.34$$

* A wooden beam 140mm wide & 240mm bend has a span of 4m determine the load that be placed at its centre to cause the beam a deflection of 10mm take 'E' as 6 GPA

Sol: Given

$$\frac{bd^3}{12} = \frac{140 \times 120}{12}$$

$$= 161280000$$

10mm

$$W = \frac{48EI(10)}{l^3}$$

$$= I(45000)$$

$$= 161280000 \times 45000$$

$$= 16128 \times 45 \times 10^7 \times 10^{-9} \; (\because 10^{-3}\text{ length})$$

$$= 725760 \times 10^{-2}$$

$$= 7257.60\,N$$

$$= 7.25760\,KN$$

$$\boxed{= 7.25\,KN}$$

1) A beam of uniform section of span 'l' is simply supported at its end it is carrying a point load of 'w' at a distance of 'l/3' from one end find the deflection of beam under the load.

sol: distance b/w load & right end $(b) = l - l/3$

$$= \frac{2l}{3}$$

$$a = l/3$$

W.K.T deflection under load.

$$\frac{wab}{6EIl} \times (l^2 - a^2 - b^2)$$

$$= \frac{w(l/3)(2l/3)}{6EIl}\left(l^2 - (l/3)^2 - (2l/3)^2\right)$$

$= 0.0165$ ...

$$= \frac{w(l/3)(2l/3)}{6EIl}\left(l^2 - l^2/9 - \frac{4l^2}{9}\right)$$

$$= \frac{w(l/3)(2l/3)}{6EIl}\left(l^2 - \frac{3l^2}{9}\right)$$

$$= \frac{w(l/3)(2l/3)}{6EIl} \times \frac{6l^2}{9}$$

$$\boxed{= 0.0165 \times \frac{wl^3}{6EI}}$$

5) A steel joist simple support over a span of 6m carries a point load 50kN at an 1.2m from left hand support. find the position & magnitude of the maximum deflection. take $EI = 14 \times 10^{12}$ N/mm²

sol $l = 6m = 6 \times 10^3$ mm.

$EI = 14 \times 10^{12}$

$a = 1.2 \times 10^3$ mm

load $= 50 \times 10^3$ N

Position & magnitude of the maximum deflection

$$x = \sqrt{l^2 - a^2}$$

W.K.T. Position of maximum deflection (or) distance b/w point of maximum deflect & left hand support

$$x = \sqrt{\frac{l^2 - a^2}{3}} = \sqrt{\frac{6 \times 10^3 - 1.2 \times 10^3}{3}}$$

$x = 3.39 \times 10^3$ mm.

magnitude of maximum deflection.

$$y_{max} = \frac{Wa}{9\sqrt{3}EIl}(l^2 - a^2)^{3/2}$$

$=$

3) A simply supported beam of span of 5m carrying a load of 30kN at a distance of 3.75m from left end, calculate the slope at a & b and deflection

Sol:

$EI = 26 \times 10^{12}\ mm^2$

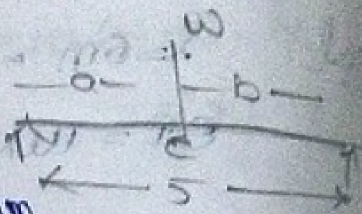
$l = 5m = 5 \times 10^3 mm$

$W = 30kN = 30 \times 10^3 N$

$a = 3.75m = 3.75 \times 10^3 mm$

$b = 5 - 3.75m$

$= 1.25m = 1.25 \times 10^3 mm$



Slope 'a'

$$\frac{dy}{dx} = \frac{Wb}{6EIl}(l^2 - b^2)$$

$$= \frac{30 \times 10^3 \times 1.25 \times 10^3}{6 \times 26 \times 10^{12} \times 5 \times 10^3}\left((5 \times 10^3)^2 - (1.25 \times 10^3)^2\right)$$

$$= \cancel{0.0115}\ 0.00113 \text{ radian}$$

Slop 'b'

$$\frac{dy}{dx} = \frac{Wa}{6EIl}(l^2 - a^2)$$

$$= \frac{30 \times 10^3 \times 3.75 \times 10^3}{6 \times 26 \times 10^{12} \times 5 \times 10^3}\left((5 \times 10^3)^2 - (3.75 \times 10^3)^2\right)$$

$$= 0.00158 \text{ radian}$$

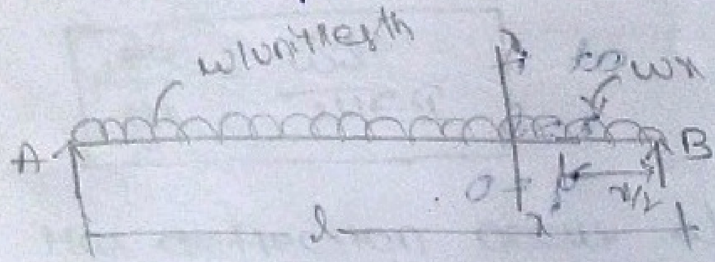
deflection:

$$y_c = \frac{Wab}{6EIL}(l^2 - a^2 - b^2)$$

$$= \frac{30\times10^3 \times 3.75\times10^3 \times 1.25\times10^3}{6\times 26\times10^{12}\times 5\times10^3}\left((5\times10^3)^2 - (1.25\times10^3)^2 - (3.75\times10^3)^2\right)$$

$= 1.62259$ radian,,

consider a simply supported beam of length l carring a U.D.L of load w/unit length as shown in fig



$R_A + R_B = wl$

$-R_B \times l + wl \times l/2 = 0.$

$-R_B \times l = \frac{wl^2}{2}$

$R_B = \frac{wl}{2}$

$R_A = wl - \frac{wl}{2}$

$= \frac{wl}{2}$

consider a section x-x at distance of x from B

$M_{xx} = -R_B x + wx \cdot x/2$

$$= -\frac{wl}{2}x + \frac{wx^2}{2}$$

$$EI\frac{d^2y}{dx^2} = -\frac{wlx}{2} + \frac{wx^2}{2}$$

Integrating above eqn

$$EI\frac{dy}{dx} = -\frac{wlx^2}{4} + \frac{wx^3}{6} + C_1 \quad \text{--- (1)}$$

again integrating

$$EIy = -\frac{wlx^3}{12} + \frac{wx^4}{24} + C_1x + C_2 \quad \text{--- (2)}$$

applying boundary condition.

at point B

$x=0$

$y=0$

$l=0$

at A

$x=l$

$y=0$.

apply Boundary condition in eqn (2)

$$0 = 0 + 0 + 0 + C_2$$

$$\boxed{C_2 = 0}$$

apply B.C (ii) (2)

$$0 = -\frac{wl^4}{12} + \frac{wl^4}{24} + C_1l + 0.$$

$$C_1l = \frac{wl^4}{12} - \frac{wl^4}{24}.$$

$$C_1 l = \frac{wl^4}{24}$$

$$C_1 = \frac{wl^3}{24}$$

$$EI\left(\frac{dy}{dx}\right) = -\frac{wlx^2}{4} + \frac{wx^3}{6} + \frac{wl^3}{24} \quad \text{--- (3)}$$

$$EIy = -\frac{wlx^3}{12} + \frac{wx^4}{24} + \frac{wl^3x}{24} \quad \text{--- (4)}$$

Max slope occur at A & B

at B, $x=0$.

$$EI\frac{dy}{dx} = \frac{wl^3}{24}$$

$$\boxed{\frac{dy}{dx} = \frac{wl^3}{24EI}}$$

Max deflection occur at $x = l/2$

$$EIy = -\frac{wl \times l^3}{96} + \frac{wl^4}{384} + \frac{wl^4}{48}$$

$$= -\frac{wl^4}{96} + \frac{wl^4}{384} + \frac{wl^4}{48}$$

$$= \frac{-4wl^4 + wl^4 + 8wl^4}{384} = \frac{5wl^4}{384}$$

$$\boxed{y = \frac{5wl^4}{384EI}}$$

A simply supported beam of span 4m is carring a U.D.L of 2kN/m over the entire span find the max slope & deflection of the beam take $EI = 80\times10^9$ N/mm²

Sol^n: $l = 4\times10^3$ mm

$w = \cancel{2}\ 8\times10^3$ mm.

$EI = 80\times10^9$ N/mm²

$$\frac{dy}{dx} = \frac{wl^3}{24EI} = \frac{2\times4\times10^3\times10^3\times(4\times10^3)^3}{24\times80\times10^9}$$

$$= \boxed{0.266\text{mm}.}$$

$$y = \frac{5wl^4}{384EI}$$

$$= \frac{5\times8\times10^3\times(4\times10^3)^4}{384\times80\times10^9} = 3333.33$$

A simply supported beam span 6m of u.d.l over a entire span if the deflection at the centre of the beam is not to exceed 4mm. find the value of load. take $E = 200 \times 10^3$ N/mm^2 & $I = 300 \times 10^6$ mm^4.

Sol: W.k.T deflection at centre

$$y_c = \frac{5wl^4}{384EI}$$

$$5wl^4 = y_c \, 384EI$$

$$w = \frac{y_c \times 384EI}{5l^4}$$

$$w = \frac{4 \times 384 \times 200 \times 10^3 \times 300 \times 10^6}{5 \times (6 \times 10^3)^4}$$

$$w = 14.22 \text{ kN/mm}$$

4) A timber beam of ~~deflection~~ span of 4.8m & is simply supported at its end; it is required to carry 45kN over the hole span find the values of breath & of the beam, if the maximum bending stress exceed to 7MPa. & maximum deflection is to 9.5mm. take $E = 10.5$ GPa.

Sol^n $l = 4.8m = 4.8 \times 10^3 mm$

$wl = 45 kN = 45 \times 10^3 N$

$\sigma_{max} = 7 MPa = 7 N/mm^2$

$b = ?$ $E = 10.5 GPa = 10.5 \times 10^3 N/mm^2$

$d = ?$

$$M = \frac{wl^2}{8} = \frac{wl \times l}{8} = \frac{45 \times 10^3 \times 4.8 \times 10^3}{8}$$

$= 27 \times 10^6 N/mm^2$

$$I = \frac{bd^3}{12}$$

$y = d/2$ (distance b/w Nutural axis & extreme layer of fibre)

Max bending stress $\sigma_{max} = \frac{M}{I} y$

$$= 7 = \frac{27 \times 10^6 \times d/2}{\frac{bd^3}{12}}$$

$$bd^2 = \frac{162 \times 10^6}{7}$$

$bd^2 = 23.14 \times 10^6$ —— (1)

we also known max deflection.

$$y = \frac{5wl^4}{384EI} = \frac{5 \times wl \times l^3}{384EI}$$

$$9.5 = \frac{5 \times (45 \times 10^3) \times (4.8 \times 10^3)}{384 \times (10.5 \times 10^3) \times \frac{bd^3}{12}}$$

$$bd^3 = \frac{74.1 \times 10^9}{9.5}$$

$$bd^3 = 7.8 \times 10^9 \text{ —— } (2)$$

(2) ÷ (1)

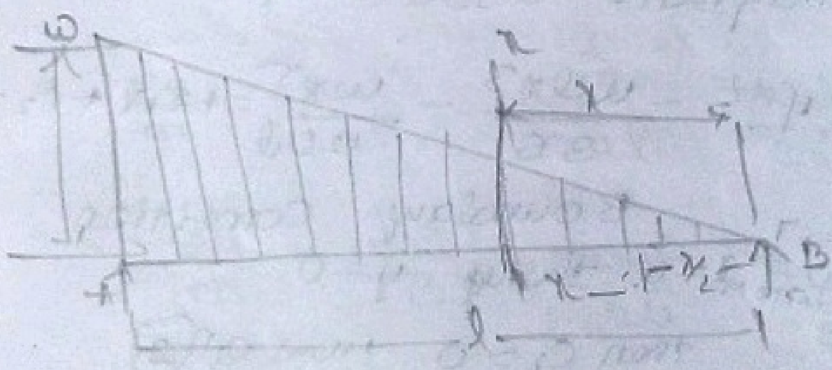
$$d = \frac{7.8 \times 10^9}{23.14 \times 10^6} = 337\text{mm}$$

sub d in eqn (1)

$$b \times (337)^2 = 23.14 \times 10^6$$

$$b = 204 \text{ mm}$$

simple supported beam with gradually varing load



$$R_A = wl/3 \quad , \quad R_B = \frac{wl}{6}.$$

consider a simply supported beam AB of length 'l' & carrying a gradually varing load from 0 ~~both~~ at B to w/unit length at A as shown in fig. from the geometry of fig the reaction force R_A & R_B values given as known consider a section. at a distance x from 'b' we known that Bending. Benmoment. is section.

$$M_{xx} = R_B \cdot x - \left(\frac{wx}{l} \times x/2 \times x/3\right)$$

$$= \frac{wlx}{6} - \frac{wx^3}{6l}$$

$$EI \frac{d^2y}{dx^2} = \frac{wlx}{6} - \frac{wx^3}{6l} \text{ ——— } (1)$$

integrating above eq

$$EI \frac{dy}{dx} = \frac{wlx^2}{12} - \frac{wx^4}{24l} + C_1 \text{ ——— } (2)$$

integratir above eqn

$$EI\, y = \frac{wlx^3}{36} - \frac{wx^5}{120l} + C_1x + C_2 = 0$$

W.k.T Boundary condition.

(i) at $x=0$ then $y=0$

then $C_2 = 0$ from eqn (3)

ii) When $x = l$, $y = 0$ value in substitute eqn. 3

$$0 = \frac{wl}{36} l^3 - \frac{w}{120l} l^5 + C_1 l$$

$$= \frac{wl^4}{36} - \frac{wl^{5\,4}}{120\cancel{l}} + C_1 l$$

$$c_1 = \frac{wl^3}{36} + \frac{wl^3}{120}.$$

$$c_1 = \frac{-7wl^3}{360}$$

Now substitue the value of c_1 in eqn (2).

$$EI\frac{dy}{dx} = \frac{wlx^2}{12} - \frac{wx^4}{24l} - \frac{7wl^3}{360} \quad \text{—— (4)}$$

consider a little consideration will show that the maximum slope will be either support a or b the slope at a = substute $x = l$ in eqn (4)

$$EI\frac{dy}{dx} = \frac{wl^3}{12} - \frac{wl^{4}}{24l} - \frac{7wl^3}{360}$$

$$EI\frac{dy}{dx} = \frac{wl^3}{12} - \frac{wl^3}{24} - \frac{7wl^3}{360}.$$

$$EIy = \frac{wl^3}{45}$$

$$y = \frac{wl^3}{45EI}$$

Now slope at B

$x = 0$ in eqn (4)

$$EIy = -\frac{7wl^3}{360}.$$

$$y_B = \frac{-7wl^3}{360EI}.$$

Now subsutite BC the value C_1.

$$EIy = -\frac{wlx^3}{36} - \frac{wx^5}{120l} - \frac{7wl^3}{360}x$$

$$y = \frac{1}{EI}\left(-\frac{wlx^3}{36} - \frac{wx^5}{120l} - \frac{7wl^3}{360}x\right) \text{---} (5).$$

for the deflection at centre of the beam, subsutite $x = l/2$ in above e^n

$$y = -0.0065\frac{wl^4}{EI}$$

W.K.T the max deflection will occure where slope of the beam is zero.

$\therefore$ eq^n 4 to zero.

$$EI\ \frac{wlx^2}{12} - \frac{wx^4}{24l} - \frac{7wl^3}{360} = 0.$$

$x = 0.519l$.

Now 'x' value in eq^n 5.

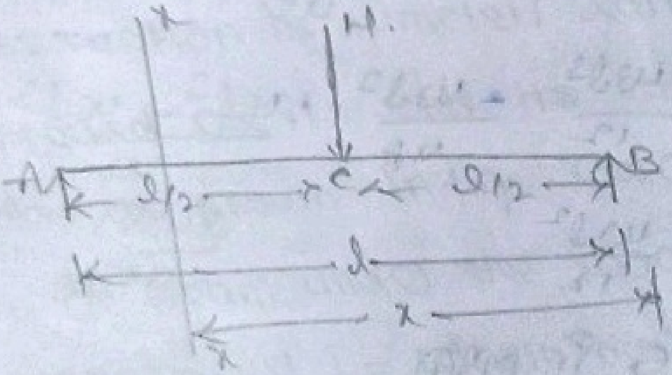
$$= -0.065\frac{wl^4}{EI}$$

(1) A simply supported beam of span 4m is carrying a triangular load varying of zero at A to 5 KN/m at B Determine max. deflection of beam take rigidity 1.25×10^4 N/m.

Macalay's method for slop & deflection

simply supported beam with a central point load

consider a S.S.B. AB of length 'l' and carrying a point load 'w' at the centre of beam 'c' as shown in fig.



$R_A + R_B = W$

$-R_B \times l + W \times l/2 = 0$

$-R_B \times l = -W \times l/2$

$$\boxed{R_B = W/2}$$

$M_{xx} = -\frac{W}{2}x + W(x - l/2)$

$EI\frac{d^2y}{dx^2} = -\frac{W}{2}x + W(x - l/2)$

integrating

$EI\frac{dy}{dx} = -\frac{Wx^2}{4} + C_1 + \frac{W(x - l/2)^2}{2}$ —— (1)

again integration.

$$EIy = -\frac{wx^3}{12} + C_1x + C_2 + \frac{w(x-l/2)^3}{6} \quad \text{--- (2)}$$

Boundary conditions:-

i) at B — ii) at (A)

$x=0$ — $x=l$

$y=0$ — $y=0$.

$l=0$

apply B.C (i) in eⁿ (2)

$C_2 = 0$.

Now apply B.C (ii) in eⁿ (2)

$$0 = \frac{-wl^3}{12} + C_1 l + 0 + \frac{wl^3}{48}$$

$$C_1 l = \frac{wl^3}{12} - \frac{wl^3}{48} \quad \frac{4wl^3 - wl^3}{48} = \frac{\cancel{3}wl^3}{\cancel{4}\times 16} =$$

$$C_1 = \frac{wl^2}{12}$$

Subsutite C_1 in eⁿ (1)

$$EI\frac{dy}{dx} = \frac{-wx^2}{4} + \frac{wl^2}{16} + \frac{w}{2}(x - l/2)^2$$

Maximum slope occure at A & B, at B

$x=0$,

apply only upto datum line

$$EI\frac{dy}{dx} = \frac{wl^2}{16}.$$

$$\boxed{\frac{dy}{dx} = \frac{wl^2}{16EI}} \rightarrow \text{maximum slope.}$$

$$EIy = -\frac{wx^3}{12} + \frac{wl^2}{16}x + \frac{w}{6}\left(x - l/_2\right)^2$$

maximum deflection occurs at $x = l/_2$.

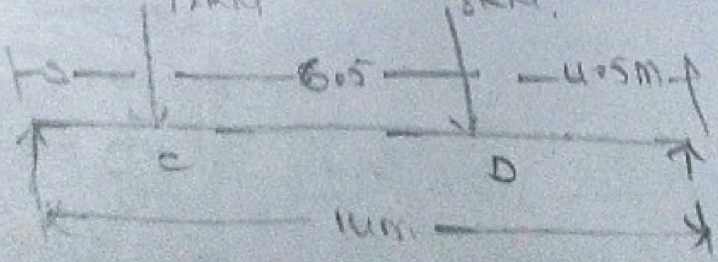
$$= -\frac{wl^3}{96} + \frac{wl^3}{32}$$

$$= \frac{-wl^3 + 3wl^3}{96} = \frac{2wl^3}{96} = \frac{wl^3}{48}$$

$$EIy = \frac{wl^3}{48}$$

$$\boxed{y_{max} = \frac{wl^3}{48EI}} \rightarrow \text{max deflection.}$$

A horizontal steel girder having uniform crosssection is 14meters long & simply supported as its end. its carries two concentrated loads as shown in fig. Calculate the deflection of beam under the loads and take. $E = 200 GPa$ & $I = 160 \times 10^4 mm^4$.



$R_A + R_B = 20 kN$

$-R_B \times 14 + 8(9.5) + 12(3) = 0$

$-14 R_B = -112$

$R_B = 8 kN$

$R_A = 20 - 8$

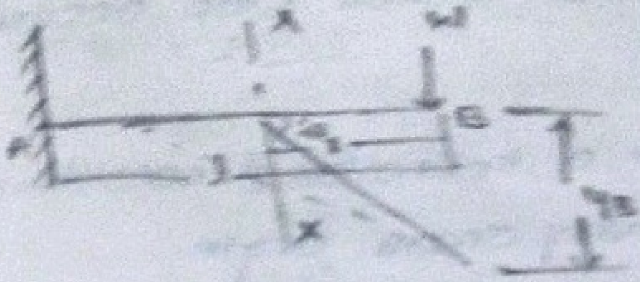
$R_A = 12 kN$

$EI \frac{d^2y}{dx^2} = -\frac{w}{2}x + w(x - l/2)$

$= -\frac{12}{2}x + 12(x - 14/2)$

Deflection of cantilever beam:

Cantilever with a point load at it's free end:



Consider a cantilever beam of 'AB' length l carrying a point load 'W' at the free end as shown in fig.

Consider a section 'X' at a distance 'x' from the free end B. We know that bending moment at the section M_x

$M_x = -Wx$

$EI \frac{d^2y}{dx^2} = -Wx$ —— (1)

Integration

$EI \frac{dy}{dx} = \frac{-Wx^2}{2} + C_1$ —— (2)

applying the boundary condition.

When $x = l$, $\frac{dy}{dx} = 0$

$C_1 = \frac{Wl^2}{2}$

Subsitute c_1 in e^n ③.

$$EI\frac{dy}{dx} = -\frac{wx^2}{2} + \frac{wl^2}{2} \longrightarrow ③ \xrightarrow{} \text{deflection.}$$

This is the requried e^n for the slope of any section and maximum slop accures at free end.

i = for the angle of inclination & considering i = tan i for a very small angle.

for the maximum slope subsutite $x=0$ and in e^n 3

$$EI\,i_B = \frac{wl^2}{2}$$

$$i_B = \frac{wl^2}{2EI} \longrightarrow \text{angle} \quad ⓐ$$

integrating e^n ③ again

$$EI = -\frac{wx^3}{6} + \frac{wl^2x}{2} + c_2 \text{ ——— } ④.$$

Where c_2 is the second constant of integrat

apply the boundary condition as $x=l \cdot y=0$

$$= \frac{wl^3}{6} + \frac{wl^3}{2} + c_2.$$

$$c_2 = \frac{wl^3 + 3wl^3}{6}$$

$$c_2 = -\frac{wl^3}{3} \text{ (due to downward deflection.}$$

Subsuting the value c_2 vale in e^n ④.

$$EI = \frac{Wx^3}{6} + \frac{Wl^2x}{2} - \frac{Wl^3}{3}$$

This is the required eqⁿ for any section the deflection

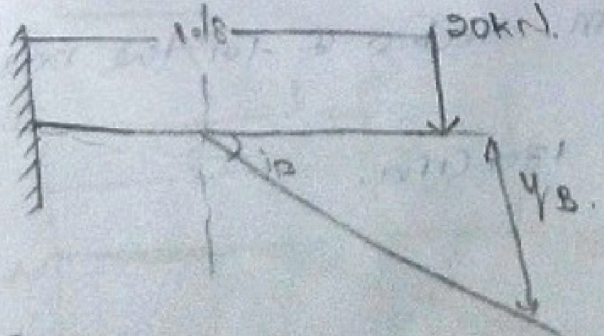
so substitute $x=0$ in eqⁿ — ⑤

$$EI = -\frac{Wl^3}{3}$$

$$y_B = -\frac{Wl^3}{3EI}$$

$$= \frac{Wl^3}{3EI}$$ (due to the clockwise direction)

A cantilever beam with 120mm wide and 150mm deep is 1.8m long. determine the slope and deflection at the free end of the beam when it is carries a point load of 20kN at it's free end. take E is 200GPa



$$I = \frac{Bd^3}{12}$$

$$I = \frac{120 \times 150^3}{12}$$

$$I = 33750000 \times 10^6$$

$$= 33.75 \times 10^6 \ mm^4$$

$$\theta_B = \frac{Wl^2}{2EI}$$

$$= \frac{(20 \times 10^3)(1.8 \times 10^3)^2}{(2 \times 200 \times 10^3)(33.75 \times 10^6)}$$

$$= 0.0048 \quad 0.003$$

Deflection:

$$EI\frac{dy}{dx} = \frac{Wx^3}{6} + \frac{Wl^2x}{2} - \frac{Wl^3}{3}$$

$$EI = \frac{20\times10^3(120)^3}{6} + \frac{20\times10^3\times1.8\times10^3\times120}{2}$$

$$- \frac{20\times10^3\times1.8\times10^3}{3}$$

$$y_B = \frac{Wl^3}{3EI}$$

$$= \frac{20\times10^3\times(1.8\times10^3)^3}{3\times200\times10^3\times33.75\times10^6}$$

$$= \cancel{0.0002}\ 5.76.$$

2) A cantilever beam of 160mm width & 240mm deepth of 1.5m long. What load can be placed at the free end of cantilever, if its deflection under the load is not to exceed 4.5mm (y_B) take E for the beam materials as 180 GPa.

$$y_B = \frac{Wl^3}{3EI}$$

Sol: Given

160 mm width

240mm deepth

$y_B = 4.5\,mm \quad l = 1.5 \times 10^3\,mm.$

$E = 180 \times 10^3\,mm$

$W = ?$ $\qquad I = \frac{Bd^3}{12}$

$y_B = \frac{Wl^3}{3EI}$ $\qquad = \frac{160 \times 240^3}{12}$

$= 184.32 \times 10^6\,mm$

$y_B \times 3EI = Wl^3$

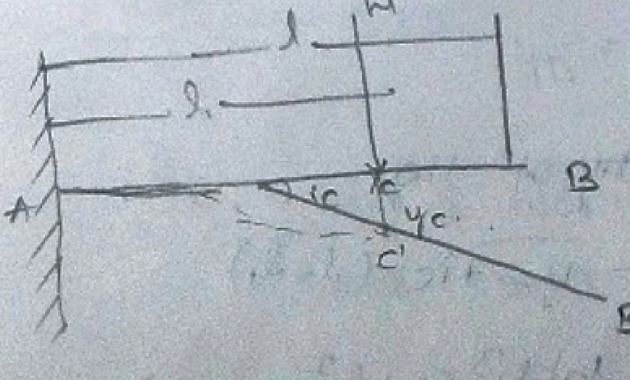
$\frac{y_B \times 3EI}{l^3} = W$

$\frac{4.5 \times 3 \times 180 \times 10^3 \times 184.32 \times 10^6}{(1.5 \times 10^3)^3}$

$= 132710.400\,kN$

F13

Cantilever beam with a point load not at free end:



consider a cantilever 'AB' of length 'l' and carring a point load 'W' at a distance 'l_1' from fixed end. as shown in above. fig

A little consideration will show that the portion 'AC' will bend into AC' due while the portion CB remain stright and displaced C'B' as shown in above fig.

$M_x = Wl_1$

$$EI \frac{d^2y}{dx^2} = -Wl_1 \text{ ——— } (1)$$

integrating above eqn

$$EI \frac{dy}{dx} = .$$

from the geometry of fig

$$y_B = y_c + i_c \cdot (l - l_1)$$

$$= \frac{Wl_1^3}{3EI} + \frac{Wl_1^2}{2EI} \cdot (l - l_1)$$

$l_1 = l/2 \cdot y_B = W/3EI (l/2)^3 + \frac{W}{2EI} (l/2)^2 \times l/2.$

$$y_B = \frac{5Wl^3}{48EI}$$

2m long

A cantilever beam carries a point of 20 kN at a distance of 2m from the fixed end. determine & slope & deflection, at the free end of cantilever take $EI = 8 \times 10^{12} N/mm^2$

Sol: $l = 3m = 3 \times 10^3 mm$ $\quad W = 20 \times 10^3 N.$

$l_1 = 2 \times 10^3 mm$

from

$EI = 8 \times 10^{12} N/mm^2.$

$$i_B = \frac{Wl^2}{2EI} = \frac{20 \times 10^3 \times 2 \times 10^3 mm}{2 \times 8 \times 10^{12}}.$$

$i_C = 0.005$ Radians.

$$y_C = \frac{Wl_1^3}{3EI} =$$

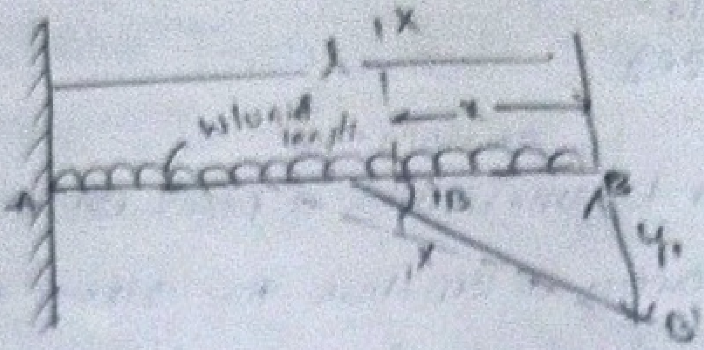
$y_B =$

$$y_B = \frac{Wl_1^3}{3EI} + \frac{Wl_1^2}{2EI}(l - l_1)$$

$$= \frac{20 \times 10^3 \times (2 \times 10^3)^3}{3 \times 8 \times 10^{12}} + \frac{20 \times 10^3 (2 \times 10^3)^2}{2 \times 8 \times 10^{12}} (1 \times 10^3)$$

$= 11.7 mm.$

Cantilever with uniformly distributed load.



consider a cantilever of AB length 'l' and carring a UDL of w/unit length as shown in fig & also consider section 'x' at a distance 'x' from the free end B

$M_x = \frac{wx^2}{2}$

$EI \frac{d^2y}{dx^2} = \frac{wx^2}{2}$ ——— (1)

integrating

$EI \frac{dy}{dx} = \frac{wx^3}{6} + C_1$ ——— (2).

Where 'C_1' is constant of integration.

W.K.T when $x = l$, $\frac{dy}{dx} = 0$. Substitu...

$C_1 = -\frac{wl^3}{6}$

Subsitute C_1 in eq (2)

$EI \frac{dy}{dx} = -\frac{wx^3}{6} + \frac{wl^3}{6}$ ——— (3).

This is the required eq (+ve due to
for the slope at any section) sagging)

W.K.T the maximum deflection occurs at free end B.

Integrating above eq^n.

for maxim slope

$$EI y = -\frac{Wx^4}{24} + \frac{Wl^3 x}{6} + C_2 \text{ ——— (4)}.$$

$x = 0$ in eq^n 3

$$EI\, i_B = \frac{Wl^3}{6}$$

$$\boxed{i_B = \frac{Wl^3}{6EI}}$$

apply Boundary conditions when $x = l$

then $y = 0$.

$$C_2 = -\frac{Wl^4}{24} + \frac{Wl^4}{6}$$

$$C_2 = \frac{3Wl^4}{24} = \frac{Wl^4}{8}$$

Substitute C_2 in eq^n (4)

$$EIy = \frac{Wx^4}{24} + \frac{Wl^3 x}{6} + \frac{Wl^4}{8}$$

apply boundary co. $x = 0$.

$$EIy = \frac{Wl^4}{8} \text{ for the deflection.}$$

$$\boxed{y_B = \frac{Wl^4}{8EI}}$$

(max. deflection in clockwise)

A cantilever beam 2 m long is subjected to uniformly distributed load of 5kN per meter over it's entire length find the slope & deflection of cantilever beam take $EI = 2.5 \times 10^{12} mm^2$

Sol^n Given

$l = 2 m \times 10^3 mm$

$W = 5 \times 10^3 N = 5 \times 10 \times 10^3$

$EI = 2.5 \times 10^{12} mm^2$

$$i_B = \frac{Wl^3}{6EI}$$

$$i_B = \frac{10 \times 10^3 \times (2 \times 10^3)^3}{6 \times 2.5 \times 10^{12}}$$

$= 5.3$

deflection

$$y_B = \frac{Wl^4}{8 \times EI} = \frac{10 \times 10^3 \times (2 \times 10^3)^4}{8 \times 2.5 \times 10^{12}}$$

$y_B = 8000 mm.$

2) A cantilever beam 100mm wide & 80mm deepth is projected 2m from. the wall calc the UDL which the beam should carry if the deflection of beam should not exceed 3.5 mm.

take $E = 200$ GPa.

Sol: Given

width = 100mm

depth = 180mm

$y_B = 3.5$mm

$$I = \frac{Bd^3}{12}$$

$$= \frac{100 \times 180^3}{12}$$

$$= 48.6 \times 10^6 mm^4$$

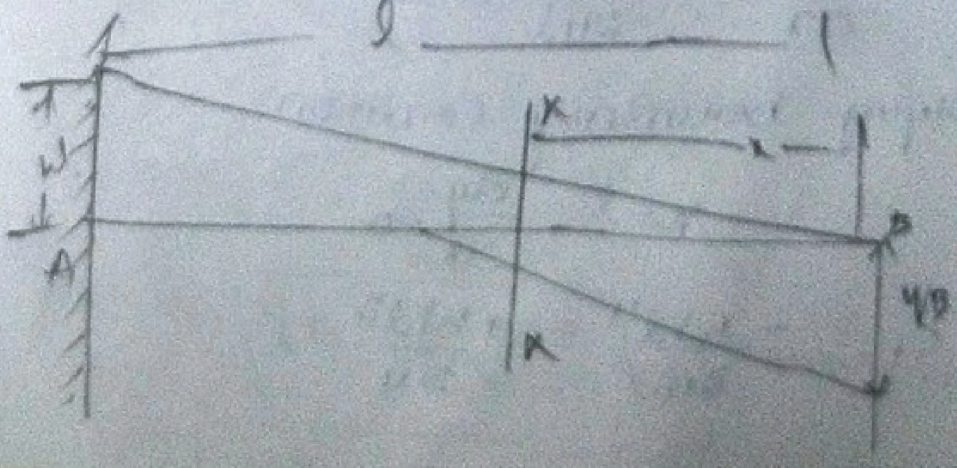
$$y_B = \frac{wl^4}{8EI}$$

$$\frac{y_B \times 8 \times EI}{l^4} = w$$

$$= \frac{3.5 \times 8 \times 200 \times 10^3 \times 48.6 \times 10^6}{(2 \times 10^3)^4}$$

$$\boxed{w = 17.01 N/mm}$$

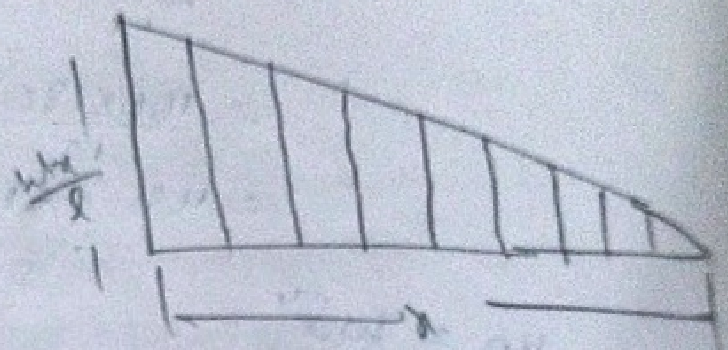
Cantilever beam. gradually varying load.



consider a Cantilever beam of AB length ~~load~~ ~~UDL~~ UVL from at B as shown in fig. also consider a section 'x' at a distance x from the free end.

Moment at x =.

By the rule of triangular.



$$M_x = -\frac{1}{2} \times \frac{Wx}{l} \times x \times \frac{x}{3}.$$

$$= \frac{Wx^2}{2l} \times \frac{x}{3}$$

$$= -\frac{Wx^3}{6l}.$$ (-ve due to hogging)

$$EI \frac{d^2y}{dx^2} = -\frac{Wx^3}{6l}.$$

Integration on both

$$EI \frac{dy}{dx} = -\frac{Wx^4}{24l} + C_1 \text{ ——— (1).}$$

applying boundary Condition

$$x = l \quad \frac{dy}{dx} = 0$$

$$C_1 = -\frac{Wl^4}{24l} = +\frac{Wl^3}{24}$$

Subst C_1 in eqn (1)

$$EI\frac{dy}{dx} = -\frac{wx^4}{24l} + \frac{wl^3}{24} \quad \text{--- (2)}$$

i_B ~~ate~~ put $x=0$

$$\boxed{i_B = \frac{wl^3}{EI\,24}}$$ Slope.

$$\frac{24 \times 5}{120}$$

Integrate the eqn 2 again

$$EIy = -\frac{wx^5}{120l} + \frac{wl^3x}{24} + C_2 \quad \text{--- (3)}.$$

apply Bound

$y=0 \quad x=l$

$$= -\frac{wl^{\cancel{5}\,4}}{120\cancel{l}} + \frac{wl^4}{24} + C_2$$

$$= \frac{\cancel{4}wl^4}{\cancel{120}\,30}$$

$$C_2 = -\frac{wl^4}{30}$$

Substitute C_2 in eqn (3)

$$EIy = -\frac{wx^5}{120l} + \frac{wl^3x}{24} + \frac{wl^4}{30}.$$

$= \quad x=0$

$$EIy = -\frac{wl^4}{30}$$

$$\boxed{y_B = -\frac{wl^4}{30EI}}$$ for the deflection

(-ve due to force acting downward direction)

7) A Cantilever of 2m Span carries a triangular load of 0 intensity at the free end. & 100kN/m. at the fixed end. determine the slope & deflection, at the free end take $I = 100 \times 10^6 mm^4$. & $E = 200GPa$

Sol: Given

$$l = 2 \times 10^3 mm$$

$$w = 100 \times 10^3 N/m$$

$$I = 100 \times 10^6 mm^4$$

$$E = 200 \times 10^3 GPa$$

$$\text{Slope} = \frac{wl^3}{24EI}$$

$$= \frac{100 \times 10^3 \times (2 \times 10^3)^3}{24 \times 200 \times 10^3 \times 100 \times 10^6}$$

$$= 0.016$$

$$\text{deflection} = -\frac{wl^4}{30 \times EI}$$

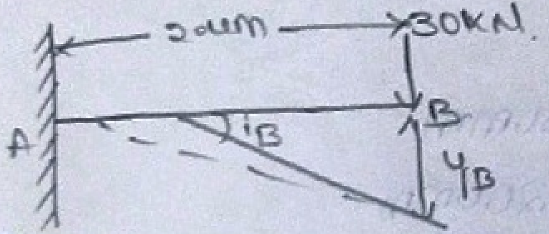
$$= -\frac{100 \times (2 \times 10)^4}{30 \times 200 \times 10^3 \times 100 \times 10^6}.$$

$$= 2.7.$$

1) A Cantilever 2.4m Carries a point load of 30kN at it's free end. find the slope & deflection of the cantilever under the load. take $EI = 25\times10^{12}\ N/mm^2$.

2) A cantilever beam 150mm wide & 200 deep projected 1.5m out of a wall & find the slope & deflection a cantilever at the free end when it carries a point load of 150 kN at it's free end. take $EI = 200GPa$.

1A



Given,

$l = 2.4m = 2.4\times10^3 mm.$

$W = 30kN = 30\times10^3 N$

$EI = 25\times10^{12} N/mm^2$

$$\text{slope} = \frac{Wl^2}{2EI}$$

$$= \frac{30\times10^3\times(2.4\times10^3)^2}{2\times25\times10^{12}}$$

$i_B = 0.00346$ radian.

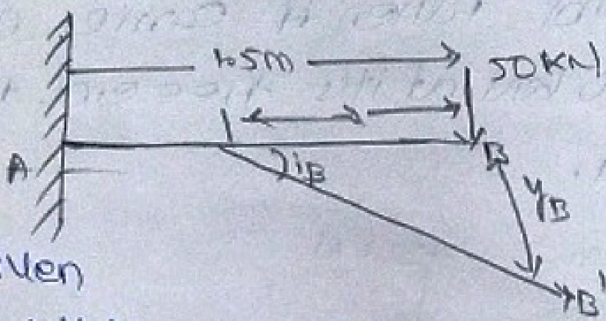
Deflection

$$= \frac{Wl^3}{3EI}$$

$$= \frac{30\times10^3 \times (2.4\times10^3)^3}{3\times25\times10^{12}}$$

$$= 5.296\,mm$$

2A



Given

Width = 150mm

deepth = 200mm

$l = 1.5 = 1.5\times10^3\,mm$

$W = 50kN = 50\times10^3\,N$

$E = 200GPA = 200\times10^3\,mm$

$$I = \frac{BD^3}{12} = \frac{150\times200^3}{12} = 1\times10^8$$

$$\text{Slope} = \frac{Wl^2}{2EI}$$

$$= \frac{50\times10^3\times(1.5\times10^3)^2}{2\times2000\times10^3\times10^8}$$

$$= 0.00281\,\text{radian.}$$

Deflection

$$= \frac{Wl^3}{3EI}$$

$$= \frac{50\times10^3\times(1.5\times10^3)^3}{3\times200\times10^3\times10^8}$$

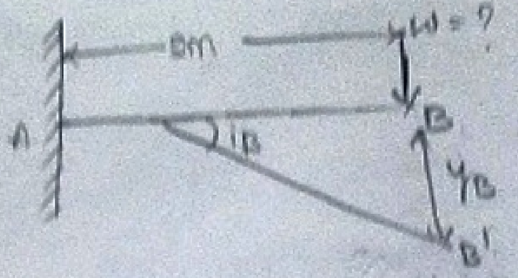
$= 2.81250mm.$

3) A cantilever beam of 120mm & 180 deep. is 2m long find the maximum load which can be placed at the free end the deflection of the cantilever at its free end should not exceed 5mm. take E as 200GPa.

4) A cantilever beam of length 1.8m is carring is a UDL of 10kN/m ~~per~~ 1 on its entire length. What is the slope & deflection of a beam at its free end. Take $EI = 3.2\times10^{12} N/mm^2$

5) A cantilever beam 2.5m span. Carries a load which is gradually Varing from 0 at free end to 200kN/m. over the fixed end find the deflection of the free end take the $EI = 160\times10^{12} N/mm^2$

3A



Given

$l = 2m = 2 \times 10^3$ mm

$W = ?$

Width = 120 mm

deepth = 180 mm

$E = 200GPa = 200 \times 10^3 \text{mm}^2$

$y_B = 5$ mm

$I = \frac{BD^3}{12}$

$= \frac{120 \times 180^3}{12}$

$I = 58.32 \times 10^6$

$y_B = \frac{Wl^3}{3EI}$

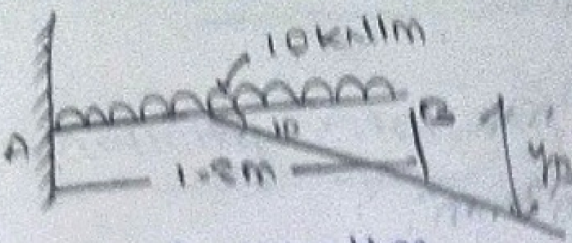
$y_B \times 3 \times EI = Wl^3$

$\frac{y_B \times 3 \times EI}{l^3} = W.$

$\frac{5 \times 3 \times 200 \times 10^3 \times 58.32 \times 10^6}{(2 \times 10^3)^3}$

$W = \cancel{21.870}\ 21870.$

4)



Given = $w = 10\,kN/m$

$= 10 \times 1.8 = 18\,kN.$

$= 18 \times 10^3\,N.$ $l = 1.8 \times 10^3\,mm.$

$EI = 3.2 \times 10^{12}\,N/mm^2$

Slope

$= \frac{wl^3}{6EI}$

deflection

$\frac{wl^4}{8EI}.$

$$= \frac{18 \times 10^3 \times (1.8 \times 10^3)^3}{6 \times 3.2 \times 10^{12}}.$$

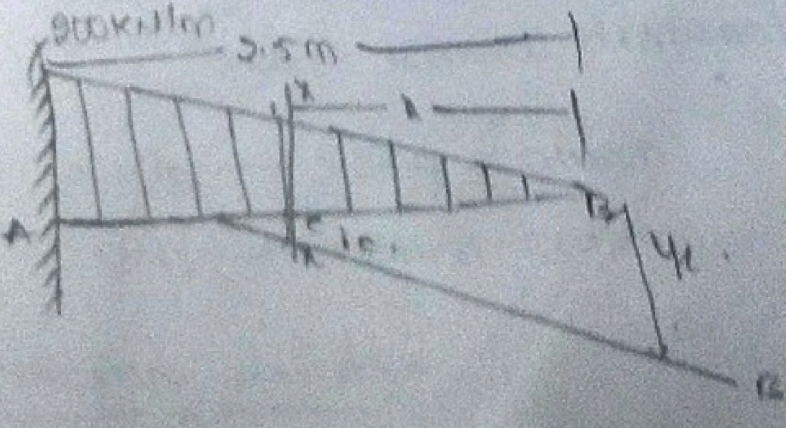
Slope $= 5.46$

Deflection

$$= \frac{18 \times 10^3 \times (1.8 \times 10^3)^4}{8 \times 3.2 \times 10^{12}}$$

$= 7381.125\,mm.$

5)



Sol: Given

$\lambda = 2.5\times10^3 mm$

$EI = 160\times10^{12} mm^2$.

$W = 200 KN/m$.

$$y_B = -\frac{Wl^4}{30EI}$$

$$= -\frac{200\times10^3\times(2.5\times10^3)^4}{30\times160\times10^{12}}$$

$y_B = 1.627 KN/mm^2$

UNIT-IV (795-820)

Elasticity Stability of column:-

Strut:- It is an structural member. Which is subjected axial compressive load. then it is called strut.

~~A structural~~ .

A strut may be horizontal, vertical, inclined. but a vertical strut we used in bulding (or) frames called column.

Types of column:-

1) long column
2) short column.

Bucking

slenderness ratio:-

$$R = \frac{\text{effective length}}{\text{least lateral dimension}}$$

$R > 12$ — long column

$R < 12$ — short column.

Effective length:-

effective length of column is defined as high height b/w the buckle. Column.

Euler's column theory:

He propersed on the long column. under the compressive load.

assumption of in the Euler's column ther

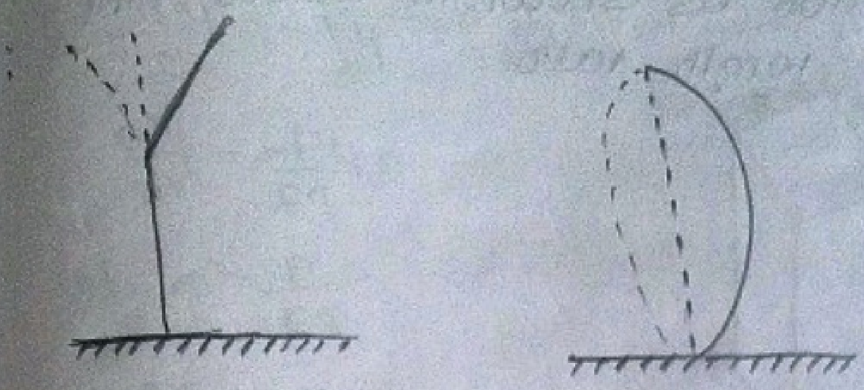
The following assumptions of Euler's.

* Initially the column is perfectly stright and load applied is axial load.
* The cross section of column is uniform through out its length.
* The column is perfectly elastic, homo genious & isotrophic & thus obey huck's law.

same properties at throughout the entire length.

* The length of the column is very large. as compared it's cross sectional dimension.
* Shorting of a column due to direct compression is negeclected.
* The failure of column occurs due to buckling alone

Sign convention:-



(a) Positive

(b) negative.

i) A moment which tends to bend the column with convexity towards it's initial central line as shown in fig (a). It consider as positive.

ii) A moment which tends to bend the column with its concavity towards it's initial central line as shown in fig (b) taken as negative.

Different end conditions of columns:-

They are four conditions according to the Euler's column theory.

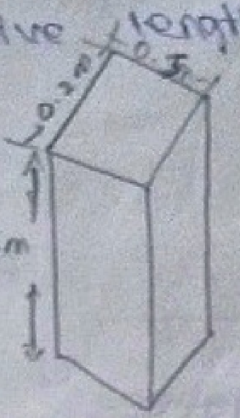
1) Both ends hinged.

2) Both ends are fixed.

3) One end is fixed other end is hinged

4) One end is fixed other end is free.

i) A rectangular column, fixed at both ends as dimension as shown in below. find effective length ratio

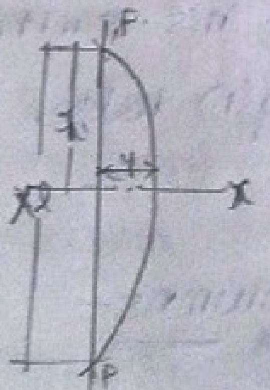


$R = \frac{6^3}{0.2} = \frac{3}{0.1} = 30 > 12$ it is long column

Column with both ends hinged.

critical load determination of column with different end condition

Both ends are hinged:-



consider a column of length 'l' which is subjected a comprisive load. The column is higed at both ends. let us consider section xx' with is at a distance of 'x' from the top end. The deflection at a section 'xx'' is 'y'

$M_x = -Py$

$EI \frac{d^2y}{dx^2} = -Py$

$\frac{d^2y}{dx^2} = \frac{-P}{EI}y$

let $\alpha^2 = \frac{P}{EI}$

$\frac{d^2y}{dx^2} = -\alpha^2 y$

$\frac{d^2y}{dx^2} + \alpha^2 y = 0$ —— (1)

The general solution.

$y = A\sin\alpha x + B\cos\alpha x$ —— (2)

apply Boundary condition

i) $x = 0, y = 0$ in eq (2)

$B = 0$

at $x = l$, $y = 0$ substitute in eq (2).

$0 = A\sin\alpha l$

$\sin\alpha l = 0$

$\alpha l = \pi$

S.O.B.S

$\alpha^2 l^2 = \pi^2$

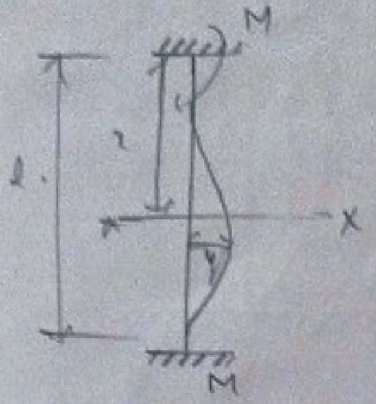
$\alpha^2 = \frac{\pi^2}{l^2}$

$\frac{P}{EI} = \frac{\pi^2}{l^2}$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

The above eqⁿ is the.
Critical load for Both ends are hinged.

Column with fixed at Both ends:-



consider a column of length 'l' fixed at both ends & subjected to a compressive load. let us consider a section 'xx' at a distance of 'x' from top end. The deflection of 'xx' is 'y'. 'M' is the moments at fixed end

$M_{xx} = -Py + M$

$EI \frac{d^2y}{dx^2} = -Py + M$

$\frac{d^2y}{dx^2} = \frac{-P}{EI} y + \frac{M}{EI}$

$\alpha^2 = \frac{P}{EI}$

$\frac{d^2y}{dx^2} = -\alpha^2 y + \frac{M}{EI}$

$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{M}{EI}$ ——— (1)

The General Solution

$y = A\sin\alpha x + B\cos\alpha x + \frac{M}{P}$ ——— (2)

$y' = A\alpha\cos\alpha x - B\alpha\sin\alpha x$ ——— (3)

Apply Boundary conditions

$x=0,\ y=0$ in eqn (2)

$0 = A\sin 0 + B\cos 0 + M/P$

$\boxed{B = -M/P}$

Apply B.C, $x=0$ $y'=0$ in eqn (3).

$0 = A\alpha\cos 0 - B\alpha\sin 0 = 0$.

$A\alpha = 0$

$y = 0 + (-M/P)\cos\alpha x + M/P$

$y = M/P(1-\cos\alpha x)$

ii) at $x = l$, $y = 0$.

$0 = M/P(1-\cos\alpha l)$

$M/P - M/P\cos\alpha l = 0$.

$\frac{M}{P}\cos\alpha l = \frac{M}{P}$

$\cos\alpha l = 1$

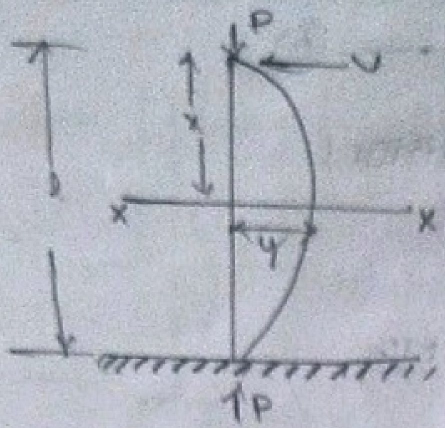
$\alpha^2 l^2 = \cancel{?}\pi^2$

$\alpha^2 l^2 = 4\pi^2$

$\frac{P}{EI} l^2 = 4\pi^2$

$\therefore P = \frac{4\pi^2 EI}{l^2}$

3) column fixed at one end and another end is hinged:-



let us consider a column of length 'l' fixed at one end and other end is hinged. consider a section 'xx' at a distance 'x' from the hinged force end and deflection about xx is 'y'. let 'V' be the horizontal load at the pinned support.

$M_{xx} = -Py - Vx.$

$EI \frac{d^2y}{dx^2} = -Py - Vx.$

let $\alpha^2 = \frac{P}{EI}$

$\frac{d^2y}{dx^2} = -\alpha^2 y - \frac{Vx}{EI}$ —— 1

$\frac{d^2y}{dx^2} + \alpha^2 y = -\frac{Vx}{EI}$ —— ①

General solution for the above eqn

$y = A\sin\alpha x + B\cos\alpha x - \frac{Vx}{P}$ —— ②.

apply the boundary conditions

i) at $x=0$, $y=0$. sub in eq (2).

$B=0$. put in eq (2)

$$\boxed{y = A\sin\alpha x - \frac{Vx}{P}} \quad (3)$$

ii) at $x=l$, $y=0$ in

$$0 = A\sin\alpha l - \frac{Vl}{P}.$$

$$\boxed{A\sin\alpha l = \frac{Vl}{P}} \quad (4)$$

iii) find y' at $x=l$, $y=0$.

from eq (3).

$$y' = A\alpha\cos\alpha x - V/P.$$

Now sub boundary conditions.

$$0 = A\alpha\cos\alpha l - \frac{V}{P}.$$

$$\boxed{A\alpha\cos\alpha l = \frac{V}{P}} \quad (5)$$

divide eq $4 \div 5$

$$\frac{1}{\alpha}\tan\alpha l = l$$

$$\tan\alpha l = \alpha l$$

$$\boxed{\alpha l = 4.5}$$

$$\alpha^2 l^2 = 20.25$$

$$\frac{P}{EI} l^2 = 20.25$$

$$\boxed{P = \frac{20.25 EI}{l^2}}$$

$$\boxed{P = \frac{2.05\pi^2 EI}{l^2}}$$

1) A column hows diameter 10cm and 3m long both ends are hinged find the buckling load that this column can carry $E = 2\times10^6$ kg/cm²

Sol: dia = 10cm

$l = 3m = 300cm$

$$I = \frac{\pi d^4}{64}$$

$$I = \frac{\pi(10)^4}{64}$$

$$I = 490.874$$

$$EI = 9.817\times10^9$$

$$P = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2\times 9.817\times10^9}{300^2}$$

$$= 1076574.516$$

2) A stright bar of alloy 1m long & 12.5mm × 4.8mm in section is mounted on a struct testing meachine load axialy till buckle. estamite maximum central deflection before the material attens its yeild point of 280MPa, take. $E = 72{,}000$ N/mm²

Sol[n]

$l = 1m = 1\times10^3 mm$.

$12.5mm \times 4.8mm$.

$P = 280MPa$.

$I = \frac{12.5\times4.8^3}{12} = 115.2mm^4$

$EI = 115.2\times72{,}000 = 8.2944\times10^6$

$P = \frac{\pi^2 EI}{l^2}$

$= \frac{\pi^2\times8.2944\times10^6}{(10^3)^2}$

$= 81.8624N$

$\sigma_{max} = \sigma_d + \sigma_b$ → direct, → bend.

$= P/A + \frac{My}{I}$

$280 = \frac{82}{(12.5\times4.8)} + \frac{82y\times2.4}{115.2}$

$280 = 1.3667 + \frac{82y\times2.4}{115.2}$

$280\times115.2 = 1.3667 + 82y\times2.4$

$\frac{280\times115.2}{2.4} = 1.3667 + 82y$.

$13440 = 1.3667 + 82y$.

$\frac{13440 - 1.3667}{82} = y$

$y = 163.88mm$

long & short column

i) for a rectangular column if $R > 12$, long column.

ii) If $R < 12$ it is short column.

iii) If length column is 20 times greater than least lateral dimension. then it is long column.

$l > 20d$ (deepth)

If the length of the column is less than least lateral dimension then it is short column

$l < 20\,d$

For circular column :-

If $l > 20D$ - long column.

If $l < 20d$ - short column.

d - diameter of .

i) A mild steel column of 50 mm dia is hinged at both of its end. find the Crippling load for the column if its length is 2.5m take $E = 200GPa$

Sol: Given

$I = \frac{\pi d^4}{64}$

dia = 50mm.

$l = 2.5m$ $E = 200\,GPa$

$$I = \frac{\pi(50)^4}{64} = ...$$

$$EI = 61.35 \times 10^6$$

$$P = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2 \times 61.35 \times 10^6}{(2.5...)^2}$$

$$= 96.88 \text{ KN}.$$

27) A section 240 x 120 x 20mm is used as 6m long column with both end fixed. What is the ~~[illegible]~~ crippling load for the column take young's modulus 200 GPa.

Sol) $l = 6m = 6 \times 10^3 mm$.

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

$$\text{section} = \underset{l}{240} \times \underset{b}{120} \times \underset{h}{20} mm$$

$$= 200 \times 10^3 mm.$$

8A $l = 1.78 = 1.7 \times 10^3 m$

$\because = 120 \times 100.$

$$P_c = \frac{2\pi^2 EI}{l^2}$$

$$I = \frac{bd^3}{12} \times 4 \times (4 \times d_1)^2$$

$$= 10 \times 10^6.$$

$$P_c = \frac{2\pi^2 \times EI}{l^2}$$

$$= \frac{2\pi^2 \times 200 \times 10^3 \times 10 \times 10^6}{(1.7 \times 10^3)^2}$$

$$= 10966.2 \times 10^3 N$$

$$= 10966.2 kN$$

Rankins formula:-

$$P_R = \frac{\sigma_c A}{1+a\left(\frac{l_e}{K_{min}}\right)^2}$$

rankins is used for short column.

R<12.

Where as Euler's formula used for long column.

P_R - rankins crippling load

σ_c - compressive stress (N/mm^2)

A — cross section of the column (mm^2)

a → Rankins constant (varies with different material)

l_e - effective length of column (depends on the end condition)

K_{min} - radius of gyration.

$$= \sqrt{\frac{I_{min}}{A}} \text{ mm.}$$

UNIT-5

Theories of Elastic failure:-

factor of saftey = $\frac{\text{ultimate stress}}{\text{allowable stress.}}$

factor of saftey is defined as the ratio of ultimate (or) yield stress to the permissible stress (or) working stress

It is used to reduce the value of stress so that a material can be stressed below the maximum stress value.

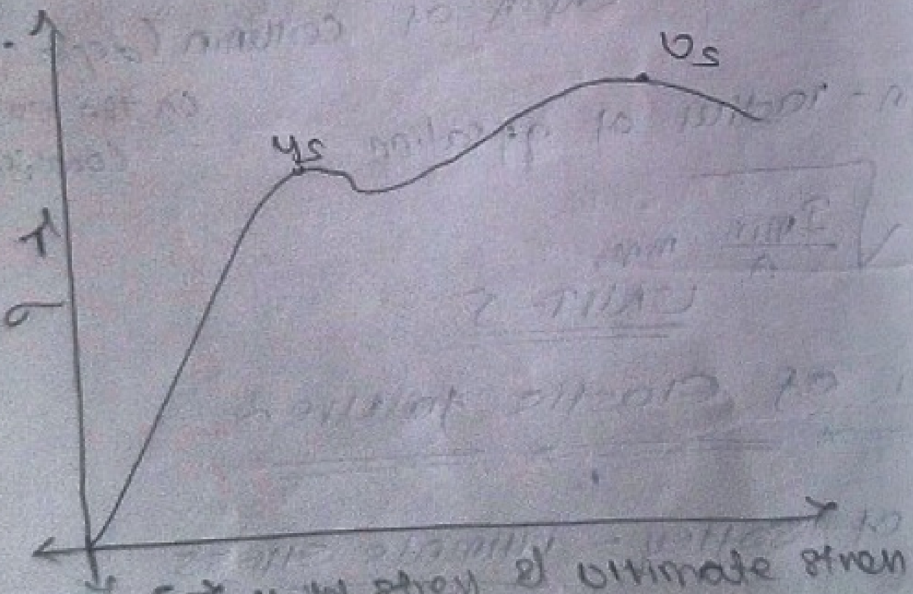
factor of saftey >1, as no units.

consider a ultimate stress for dertile material.

$$FOS = \frac{\text{Ultimate stress}}{\text{Allowable stress}}$$

considering yeild stress for dertile mater

$$FOS = \frac{\text{yeild stress}}{\text{Working stress}}$$



ε → yeild stress & ultimate stress

A steel bar 0.1m $\times$ 0.1m as ultimate stress of 100MPa subjected to a load of 300KN. find the factor of saftey.

Sol

$= \frac{\cancel{100}}{3\cancel{00}} \frac{100}{300}$

$= 0.33$

allow =
F per a
$= 300 \times 0.1 \times 0.1$
$= 3$.

allowable $= F/A = \frac{300}{0.1 \times 0.1} = 30{,}000$.

30MPa.

$= \frac{\text{ultimate stress}}{\text{Allowable stress}} = \frac{100}{30}$

$= 3.33$

$\therefore$ The materials will be safe at 300KN

Stress $= P/FOS$ $\quad$ stony $= P/FOS$

UNIT-5

Theories of failures:-

1) Maximum principle stress theory (Rankin's theory)

2) Maximum principle strain theory (state. Varient theory)

3) Maximum shear stress theory (Guest stress curve)

4) Maximum strain energy theory (Haigh theory)

5) Maximum shear strain energy theory (distorsion energy theory / Von misei. theory)

* failure is generally perceived to be fracture. or complete seperation. of a member. however failure may occuries. due to excessive deformation. (elastic (or) inelastic) (or) a varity of region reasons.

Failure modes:-

	yeilding (elastic limit)	fracture
* Excessive elastic deformation		
→ strength: twist (or) bending	plastic de-formation at room temp.	sudden fracture, brittle materials
→ Buckling	creep at ele-vated atem.	fatig (progress fracture)
→ Vibration	yeild stress is the imp design factor.	stress repeat at elevat-ed temp, stress alternat strength the imp factor

A most basic failure theory where propo-sed are tested on the few materials most theories are based on the assumption that the failure occur. when sum physical virable such as stress, strain (or) energy reaches a limiting value.

Deformation:

* elastic deformation is tempory (reversable) & involes bond strengthing.
* plastic deformation is a permant (irreversable) and involes bond breaking.
* When sum external load is applied in a body the stress & strain are produced in the body, the stress are directly. Propotional to the strain with in elastic limit.
* This means when the load is removed. the body will retain to orginal shape. Whoever if the stress produced on the body due to the application of load is beyond elastic limit, the permend de-formation occures in the body. The body said to have fail. failure doesn't mean rubcher. accorn
* According to the improved theories the failure takes place when a certain limiting value is reached by one of the following.

* The maximum principle stress theory (Rankine)
* The maximum principle strain theory (St. Venant)
* The maximum shear stress theory (Guest Tresca)
* The maximum strain energy theory (Haigh)
* The maximum shear strain energy theory (Von Mises)

In all the above case $\sigma_1 = \sigma_2 = \sigma_3$ principle stress in any complex system.

$\sigma^* =$ tensile (or) compressive stress at the elastic limit

i) The maximum principle stress theory:

according to this theory the failure of the material will occur when the maximum principal tensile stress (σ_1) in the complex system reaches the value the maximum stress at the elastic limit in simple tension (or) the minimum principle stress reaches the value of maximum stress at the elastic limit in simple compression, and this theory is known as Rankine theory.

Let in a complex three dimensional system $\sigma_1, \sigma_2, \sigma_3$ are principle stress at a point in three perpendicular direction.

The stress, σ_1, & σ_2 are tensile. & σ_3 is the compressive stress & σ_1 is more than σ_2.

* σ_t^* = tensile stress at elastic limit in a simple tension

σ_c^* = comprehive stress at elastic limit in simple compression.

∴ according to this theory the failure will occur if $\sigma_1 \geq \sigma_t^*$ in simple tension.

$|\sigma_3| \geq \sigma_c^*$ in simple compression.

∵ (11) moduler is to consider, to ignore the negetive sign.

In the given design the maximum principle stress σ_1 must no exceted the permisible stress σ_t for the given material.

Hence $\sigma_1 = \sigma_t$ where σ_t is permisible stress.

$$\sigma_t = \frac{\sigma_t^*}{\text{saffy factor}}$$

* The principle stress at a point in a elastic material are 100N/mm² (tensile), 180N/mm² (tensile) & 50N/mm² (compressive) if the stress at the elastic limit in simple tension. σ is 200N/mm². Dekrmine wheatr

the failure of material will occur according to maximum principle stress theory. if n't then determine the factor of saftey

Sol: $\sigma_1 = 100 N/mm^2$ } (tensile stress)

$\sigma_2 = 80 N/mm^2$

$\sigma_3 = 50 N/mm^2$ (compressive)

Tensile stress at elastic limit.

$\sigma_t^* = 200 N/mm^2$

as w.r.t the failure will occur

R.H.S $\sigma_1 \geq \sigma_t^*$

$100 N/mm^2 \ngeq 200 N/mm^2$

$\therefore \sigma_1$ is not equal to σ_t^* the failure of material doesn't occur. to find the factor of saftey

$$\sigma_t = \frac{\sigma_t^*}{\text{saftey factor}}$$

$\sigma_1 = \sigma_t$ saftey fact $= \sigma_t \times \sigma_t^*$

$= \cancel{100 \times 200}$

factor of saftey $= \cancel{20000}\ \frac{200}{100} = 2$

$\boxed{SF = 2}$

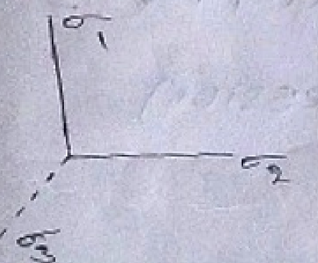
The maximum principle strain theory:

This theory is due to

according to this theory the failure will occur in a material when the maximum principle strain reaches the strain due to yeild stress. in a simple tension (or) when the minimum principle strain is the maximum. compressive strain reaches the strain due to yeild stress in a simple compression.

yeild stress is the maximum stress at elastic limit

Principle strain in the direction of Principle stress σ_1 is



$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E}$$

$$= \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

$E = \frac{\text{stress}}{\text{strain}}$ = youngs

μ is the poston's ratio.

Principle strain in the direction of principle stress

σ_3 is compression

$$e_3 = \frac{1}{E}(\sigma_3 - \mu(\sigma_1 + \sigma_2))$$

strain due to yield stress in simple tension.

$$\frac{1}{E} \times \sigma_t^* = e_t^*$$

In simple compression:

$$\frac{1}{E} \times \sigma_c^* = e_c^*$$

according to this theory the failure of material will occur take place when

$$e_1 \geq \frac{\sigma_t^*}{E} \Rightarrow \sigma_1 - \mu(\sigma_2 + \sigma_3) \geq \sigma_t^*$$

(simple tension)

$$|e_3| \geq \frac{\sigma_c^*}{E} \Rightarrow \sigma_3 - \mu(\sigma_1 + \sigma_2) \geq \sigma_c^*$$

(simple compression)

for design purpose.

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \sigma_t \left(\text{where } \sigma_t = \frac{\sigma_t^*}{S.F} \right.$$

Hence.

$$\sigma_1 = \sigma_t \text{ (or } e_1 = e_t)$$

$$|\sigma_3 - \mu(\sigma_1 + \sigma_2)| = \sigma_c \left(\text{where } \sigma_c = \frac{\sigma_c^*}{S.F} \right)$$

The principle stress at a point in a elastic material are 200N/mm² (tensile), 100N/mm² (tensile) & 50N/mm² (compressive) If the stress at the elastic limit in simple tension is 200N/mm² determine wheater the failure of material will occur according to the maximum strain theory take poison ratio as 0.3.

Sol

$\sigma_1 = 200 N/mm^2$

$\sigma_2 = 100 N/mm^2$

$\sigma_3 = 50 N/mm^2$

$e_1 \geq \frac{\sigma_t^*}{E}$

$e_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$

$e_1 = \frac{1}{E}[200 - 0.3(100 - 50)]$

$e_1 = \frac{1}{E}[200 - 0.3(50)]$

$e_1 = 1/E\,(200 - 15)$

$e_1 = 1/E\ 185$

$\frac{185}{E} \geq \frac{200}{E}$

∴ the failure of material will not occur.

Maximum shear stress theory:

This theory is due to guest theory

according to this theory the failure of a material will occur when the maximum shear stress in a material reaches the value of maximum shear stress in simple tension at the elastic limit. The maximum shear stress in the material is equal to half the difference b/w maximum & minimum principle stress.

according to this theory maximum shear stress in the material.

$= \frac{1}{2}(\sigma_1 - \sigma_3)$

maximum shear stress in simple tension at elastic limit

$= \frac{1}{2}(\sigma_t^* - 0)$

$= \frac{\sigma_t^*}{2}$

for the failure of material

$(\sigma_1 - \sigma_3) \geq \sigma_t^*$

Hence for design purpose:

$(\sigma_1 - \sigma_3) = \sigma_t$ $(60 + 30) = \sigma_t$ $= \frac{35}{70}$

$90 = \sigma_t$ 30

where σ_t is

$\sigma_t = \frac{\sigma_t^*}{SF}$

1) The principal stress at a point in a elastic material are 200 N/mm² (tensile), 100 N/mm² (tensile) & 50 N/mm² (compressive). If the stress at elastic limit is 200 N/mm² in simple tension, Determine whether the failure of material will occur (or) not according to maximum shear stress theory.

Sol:

$\sigma_1 = 200\ N/mm^2$

$\sigma_2 = 100\ N/mm^2$

$\sigma_3 = 50\ N/mm^2$

(σ_t^*) elastic limit $= 200\ N/mm^2$

failure of material,

$(\sigma_1 - \sigma_3) \geq \sigma_t^*$

$(200 + 50) \geq \sigma_t^*$

$250 \geq 200$

failure will occur in the material.

Maximum Strain energy theory

According to this theory the failure of the material will occur of material when the total strain energy per unit volume in the material reaches the strain energy per unit volume of the material at elastic limit in simple tension.

As we know strain energy $U = \frac{1}{2} \times P \times \delta L$

$$U = \frac{1}{2} \times \sigma \times A \times \delta l$$

$$U = \frac{1}{2} \times \sigma \times A \times e \times l$$

$$U = \frac{1}{2} \times V \times \sigma \times e \quad \text{---} (1)$$

$\because \sigma = P/A$

$P = \sigma \times A$

$\because e = \frac{\delta l}{l}$

$\delta l = el$

$\therefore A \times l = V$

$U \rightarrow$ Work done

$P \rightarrow$ load

$\delta l \rightarrow$ elongation.

$\therefore$ strain energy per unit volume.

$$U = \frac{1}{2} \times \sigma \times e \quad (\because \text{eq}^n (1) \div V \text{ on Both sides})$$

For three dimensional stress system. the principal stresses acting at a point are σ_1, σ_2 & σ_3 and corresponding strain are e_1, e_2 & e_3 respectively

Now

$$e_1 = \frac{1}{E}\left(\sigma_1 - \mu(\sigma_2 + \sigma_3)\right) \quad \left(\because E = \frac{\sigma}{e}\right)$$

μ = poission's ratio.

$$e_2 = \frac{1}{E}\left(\sigma_2 - \mu(\sigma_3 + \sigma_1)\right)$$

$$e_3 = \frac{1}{E}\left(\sigma_3 - \mu(\sigma_1 + \sigma_2)\right)$$

The total strain energy per unit volume in three-dimensional system is

$$U = \frac{1}{2} \times \sigma_1 \times e_1 + \frac{1}{2} \times \sigma_2 \times e_2 + \frac{1}{2} \times \sigma_3 \times e_3 \quad \text{---} (2)$$

Subsutite e_1, e_2, e_3 in eq ②

$$u = \frac{1}{2}\left[\sigma_1 \times \frac{1}{E}(\sigma_1 - \mu(\sigma_2+\sigma_3)) + \sigma_2 \frac{1}{E}(\sigma_2 - \mu(\sigma_3+\sigma_1)) + \sigma_3 \frac{1}{E}(\sigma_3 - \mu(\sigma_1+\sigma_2))\right]$$

$$u = \frac{1}{2E}\left[\sigma_1^2 - \mu \times \sigma_1(\sigma_2+\sigma_3) + \sigma_2^2 - \mu \times \sigma_2(\sigma_3+\sigma_1) + \sigma_3^2 - \mu\sigma_3(\sigma_1+\sigma_2)\right]$$

$$\Rightarrow u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Now strain energy per unit volume crrosponding to stress at elastic limit. in simple tension.

$$= \frac{1}{2} \times e_t^* \times \sigma_t^*$$ (where e_t^* is strain due to σ_t^*, $E = \frac{\sigma_t^*}{e_t^*} \Rightarrow e_t^* = \frac{\sigma_t^*}{E}$)

$$= \frac{1}{2} \times \sigma_t^* \times \frac{\sigma_t^*}{E}$$

$$= \frac{1}{2E}(\sigma_t^*)^2$$

according to this theory the failure of material will occur

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$= (\sigma_t^*)^2$$

Hence for design purpose $\cdot \sigma_t = \frac{\sigma_t^*}{\text{Saftey factor}}$

Where σ_t is

$$\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)=(\sigma_t)^2$$

1) The principal stress at a point in a elastic material are 800N/mm² (tensile) 100N/mm² (tensile) & 50N/mm² (compression). If the stress at the elastic limit in simple tension is 200N/mm². Determine whether the failure of material will occur are not according to maximum strain energy theory. If Poisson ratio as 0.3

Sol:-

$\sigma_1 = 200N/mm^2$

$\sigma_2 = 100N/mm^2$

$\sigma_3 = 50N/mm^2$

$= 81500$

Poisson ratio $= 0.3$

$$\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1) \geq (\sigma_t)^2$$

$$= 200^2+100^2+50^2-2\times0.3(200\times100-100\times50-200\times50)$$

$$\geq (200)^2$$

$$= 49500 \geq 40{,}000.$$